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A study of various non-newtonian fluid models and their applications

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Abstract
Many fluids of industrial importance are non- Newtonian. It is now generally recognized that, in real industrial applications, non-Newtonian fluids are more appropriate than Newtonian fluids, due to their applications in petroleum drilling, polymer engineering, certain separation processes, manufacturing of foods and paper and some other industrial processes. In a non-Newtonian fluid, the local shear stresses and the local shear rates in the fluid have a non-linear relation, where a proportionality constant cannot be defined. Therefore, the 'Viscosity' is not a fixed scalar but a variable. Further it is also important to note that the viscosity can be dependent on the shear rate or the time history of shear rate. Some examples are fluid substances like ketchup, custard, toothpaste, starch suspensions, paint, blood, and shampoo etc. Few widely used non-Newtonian fluid models are Power Law Model, Carreau Model, Sisko Model, Herschel-Bulkley Model, Casson, Powell-Eyring, Jeffrey & Williamson fluid model. In this paper few are discussed in detail.

Keywords: Non-newtonian fluid, viscosity, shear rate, stress and various fluid models

Introduction

Non-Newtonian Fluid Mechanics is a field of study which is growing in prominence and importance as the years progress, not least because many of the fluids one encounters in everyday life are non-Newtonian in their behavior.[1-19] Recently the blood flow of newtonion and non-newtonion fluid flow blood vessels with flexible wall[20], quantification in effective viscosity of fluid in a porous medium by Brinkman equation[21] and the oscillatory blood flow by viscosity shearing dependent model have already been studied.[22]

Materials and Methods

Classification of various non-Newtonian fluid models:

Power law model : A power-law fluid, or the Ostwald–de Waele relationship, is a type of generalized Newtonian fluid (time independent Non-Newtonian fluid) for which the shear stress, τ, is given by $\tau = K\left(\frac{\partial u}{\partial y}\right)^n$, where, K is the flow consistency index $\frac{\partial u}{\partial y}$ is the shear rate or the velocity gradient perpendicular to the plane of shear, and n is the flow behavior index (dimensionless).The quantity $\mu_{eff} = K\left(\frac{\partial u}{\partial y}\right)^{n-1}$ represents an apparent or effective viscosity as a function of the shear rate. Also known as the Ostwald–de Waele power law. This mathematical relationship is useful because of its simplicity, but only approximately describes the behaviour of a real non-Newtonian fluid. For example, if n were less than one, the power law predicts that the effective viscosity would decrease with increasing shear rate indefinitely, requiring a fluid with infinite viscosity at rest and zero viscosity as the shear rate approaches infinity, but a real fluid has both a minimum and a maximum effective viscosity that depend on the physical chemistry at the molecular level. Therefore, the power law is only a good description of fluid behaviour across the range of shear rates to which the coefficients were fitted. There are a number of other models that better describe the entire flow behaviour of shear-dependent fluids, but they do so at the expense of simplicity, so the power law is still used to describe fluid behaviour, permit mathematical predictions, and correlate experimental data.

Power-law fluids can be subdivided into three different types of fluids based on the value of their flow behaviour index:

Value of n	Type of fluid
n <1	Pseudoplastic fluids
n = 1	Newtonian fluid
n >1	Dilatant fluids (less common)

Pseudoplastic fluids: Pseudoplastic, or shear-thinning are those fluids whose behaviour is time independent and which have a lower apparent viscosity at higher shear rates, and are usually solutions of large, polymeric molecules in a solvent with smaller molecules. It is generally supposed that the large molecular chains tumble at random and affect large volumes of fluid under low shear, but that they gradually align themselves in the direction of increasing shear and produce less resistance. A common household example of a strongly shear-thinning fluid is styling gel, which primarily composed of water and a fixative such as a vinyl acetate/vinylpyrrolidone copolymer (PVP/PA). If one were to hold a sample of hair gel in one hand and a sample of corn syrup or glycerine in the other, they would find that the hair gel is much harder to pour off the fingers (a low shear application), but that it produces much less resistance when rubbed between the fingers (a high shear application).

This type of behavior is widely encountered in solutions or suspensions. In these cases, large molecules or fine particles form loosely bounded aggregates or alignment groupings that are stable and reproducible at any given shear rate. But these fluids rapidly and reversibly break down or reform with increase or decrease in shear rate. Pseudo plastic fluids show this behavior over a wide range of shear rate; however often approach a limiting Newtonian behavior at very low and very high rates of shear.

Newtonian Fluids: A Newtonian fluid is a power-law fluid with a behaviour index of 1, where the shear stress is directly proportional to the shear rate $\tau = \frac{\partial u}{\partial y}$. These fluids have a constant viscosity, μ, across all shear rates and include many of the most common fluids, such as water, most aqueous solutions, oils, corn syrup, glycerine, air and other gases. While this holds true for relatively low shear rates, at high rates most oils in reality also behave in a non-Newtonian fashion and thin. Typical examples include oil films in automotive engine shell bearings and to a lesser extent in geartooth contacts.

Dilatant Fluids: Dilatant, or shear-thickening fluids increase in apparent viscosity at higher shear rates. They are rarely encountered, but one common example is an uncooked paste of cornstarch and water, sometimes known as oobleck. Under high shear rates the water is squeezed out from between the starch molecules, which are able to interact more strongly.

Results and Discussion

The use is in a viscous coupling in which if both ends of the coupling are spinning at the same (rotational) speed, the fluid viscosity is minimal, but if the ends of the coupling differ greatly in speed, the coupling fluid becomes very viscous. Such couplings have applications as a lightweight, passive mechanism for a passenger automobile to automatically switch from two-wheel drive to four-wheel drive such as when the vehicle is stuck in snow and the primary driven axle starts to spin due to loss of traction under one or both tires.

Jeffrey Fluid Model: Jeffrey's fluid model is a linear model proposed as an extension to the Maxwell fluid model by including a time derivative of the rate of strain is given by $\tau = -\lambda_1 \frac{\partial \tau}{\partial t} + \mu_0\left(\gamma + \lambda_2 \frac{\partial \gamma}{\partial t}\right)$ where λ_2 is the retardation time that accounts for the correction of this model. The Jefferey's model has three constants *viz.* μ and two elastic parameters λ_1 and λ_2 the model reduces to linear Maxwell when $\lambda_2 = 0$ and to Newtonian when $\lambda_1 = \lambda_2 = 0$. The Jeffery's model is one of the most suitable linear models to compare with an experiment.

Herschel-Bulkley Model: The 'Herschel–Bulkley' fluid is also a generalized, non-linear model of a non-Newtonian fluids. This model combines the behaviour of Bingham and power-law fluids in a single relation. For very low strain rates, the material behaves as a very viscous fluid with viscosity ν_0. After a minimum value of strain-rate corresponding to a threshold stress τ_0, the viscosity is represented by the power law relation. For this model, the viscosity ν, is related to the shear rate γ, by the following equation: $\nu = \min(\nu_0, \tau_0/\gamma + k\gamma^{n-1})$; where, n: Power Index; k: Consistency Index; τ0: Yield stress; *v*0: viscosity at zero shear rate. If τ > τ0, the Herschel- Bulkley fluid behaves as a fluid.

Cross-Power Law Model: The Cross-Power law model is also a four-parameter model that covers the entire shear rate range. For this model, the viscosity *v*, is related to the shear rate γ.

where, *n:* Power Index; *v*0: viscosity at zero shear rate; *v*∞: viscosity at infinite shear rate; *m:* is in units of seconds. This model is reduced to the Newtonian fluid behaviour as *m* approached value zero

Bird-Carreau Model: The model was first proposed by Pierre Carreau. This is a four parameter models that is valid over the complete range of shear rates. For cases where there is significant variation from the power-law model i.e. at very high and very low shear rates, it becomes essential to incorporate the values of viscosity at zero shear *v*0 and at infinite shear *v*∞ into the formulation. For this model, the viscosity *v*, is related to the shear rate γ.

where, *a:* has a default value of 2; *k:* Relaxation time; *n:* Power Index. At low shear rate Carreau fluid behaves as a Newtonian fluid and at high shear rate as a power-law fluid. This model is mostly used for food, beverages and also blood flow applications.

Casson Fluid Model: Casson fluid is a shear thinning fluid which is supposed to have an immeasurable viscosity at zero rate of shear, a yield stress below which no movement happens, and a zero viscosity at an infinite rate of shear. Casson fluid is classified as the most popular non-Newtonian fluids which has several applications in food processing, metallurgy, drilling operations and bio-engineering operations. Casson fluid model was introduced by Casson for the prediction of the flow behavior of pigmentoil suspensions. Examples of Casson fluid are jelly, tomato sauce, honey, soup, concentrated fruit juices and human blood.

The Casson model is a basic model used in blood rheology that specifies minimum and maximum viscosities, ν_{min} and ν_{max} respectively. Beyond a threshold in strain-rate corresponding to threshold stress τ_0, the viscosity is described by a "square-root" relationship. The model is

$$\nu = \left(\sqrt{\tau_0/\gamma} + \sqrt{m}\right)^2 ; \ \nu_{min} \le \nu \le \nu_{max}$$

Eyring Model: Eyring model is a two-parameter model. The equation of Eyring model is as follow:

$$\sinh\left(\frac{\tau_{yx}}{A}\right) = -\frac{1}{B}\frac{dv_x}{dy}$$

, where A, B are the two parameters.

In Eyring model, if $\tau_{xy} \to 0$ which means very low shear forces, we have $\sinh\left(\frac{\tau_{yx}}{A}\right) \to \frac{\tau_{yx}}{A}$. Therefore, as $\tau_{yx} \to 0$, Therefore, Eyring model may be used for a fluid which shows Newtonian behaviour at low shear rates and non-Newtonian behaviour at high shear ratesthe model shows Newtonian behaviour $\tau_{yx} = -\frac{A}{B}\frac{dv_x}{dy}$, where viscosity = $\frac{A}{B}$. If τ_{yx} is very large, the model shows Non-Newtonian behaviour as shown Fig.1. Therefore, Eyring model may be used for a fluid which shows Newtonian behaviour at low shear rates and non- Newtonian behaviour at high shear rates

Maxwell Fluid Model: Maxwell fluid model is a simple linear model that combines the ideas of viscosity of fluids and elasticity of solids. Maxwell fluid model can be described by the relation $\tau = -\lambda_1 \frac{\partial\tau}{\partial t} + \mu_0 \gamma$; where τ is extra stress tensor, λ_1 is the relaxation time of the fluid, t is time, μ_0 is low shear viscocity and γ is rate of strain. This model is reducing to Newtonian if $\lambda_1 = 0$.

Conclusion

In non-Newtonian fluids, the most commonly encountered fluids are pseudoplastic fluids. The study of the boundary layer flow of Pseudoplastic fluids is of great interest due to its wide range of application in industry such as extrusion of polymer sheets, emulsion coated sheets like photographic films, solutions and melts of high molecular weight polymers, etc. The Navier Stokes equations alone are insufficient to explain the rheological properties of fluids. Therefore, rheological models have been proposed to overcome this deficiency. To explain the behaviour of pseudoplastic fluids many models have been proposed like the power law model, Carreaus model, Cross model and Ellis model, Maxwell fluid model, Williamson fluid model, upper convected Maxwell model, Jeffery fluid model etc. The boundary layer flow of a Williamson fluid over a stretching sheet Stretching sheet flows are of great importance in many engineering applications like extrusion of a polymer sheet from the die, the boundary layer in liquid film condensation processes, emulsion coating on photographic filmsetc. Herschel-Bulkley fluids include both shear thinning and shear thickening materials. The practical examples of such materials are greases, colloidal suspensions, starch pastes, tooth pastes, paints, and blood flow in an artery. The main advantage of Ellis equation is that it predicts the Newtonian behavior at small shear stresses and the power law behavior at large shear stresses. This advantage of Ellis model enables it to correctly reflect the rheological behavior of typical polymeric fluids. Carreau fluid is a type of generalized Newtonian fluid where viscosity depends upon the shear rate, The Carreau viscosity model is useful in describing flow behavior of fluids in the high shear rate region. Casson fluid model is a non-Newtonian fluid with yield stress which is widely used for modeling blood flow in narrow arteries. Many researchers have used the Casson fluid model for mathematical modeling of blood flow in narrow arteries at low shear rates.

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