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Estimating spatial structure of migration in India using saturated log linear models

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Abstract
Over the course of time, it is found that the major problems associated with population growth include differential impacts of migration among various regions, communities, city sub-urban area, etc. Indirect estimation becomes a better approach to study migration for the populations with inadequate or unreliable data. Log linear models have, earlier been discussed by Willekens (1983) [29] and Rogers (1975, 1995) [18, 23] to describe spatial interactions among populations.
The present study is comparison of inter-regional migration estimated from nested models. The saturated model explains inter-regional migration in a simpler form whereas; the Quasi Independence model proposes a rational, though coarse, procedure of estimation. Twenty-three percent of the total inter-regional out-migrants originate from Eastern region, which includes Bihar, Odisha and West Bengal. The study also indicates that there are multiple factors behind this dependency of inter-regional migration in India.

Keywords: Population growth, migration, inter-regional migration, indirect estimation, Log linear models, spatial interactions

Introduction

Over the course of time, most of the demographers dealt with single-region populations and their growth is assumed to be undisturbed by or closed to migration. Yet many of the major problems associated with the growth and development of a population revolves around differential impacts of migration among various regions, communities, city sub-urban area, etc. Migration flows involve multiregional setup where the migration from the place of origin affects population at multiple destinations, reducing population from origin and adding to population at regions of migration.
The methods of indirect estimation of migration used to be a device on missing data issues with characters changing with time but the work in last two decades have shifted on developing migration models based on theory of statistical inference to approximate parameters. Some of these models explain age patterns of migration while some explain spatial distribution of migration (Rogers, 1999) [24].
The distribution of age specific rates in different population usually vary in some pre-described limits, also, the rates of one age group affect the rates in the next age group and so on. This orderliness of interdependencies and relationships is studied and hence expressed by means of mathematical expressions known as model schedules. Model schedules are parameterized functions based on different patterns observed in the populations other than the population under study and hence estimating the population with incomplete data. The imposition of observed regularities in both the age and spatial patterns of interregional migration to "repair" unreliable data on territorial mobility holds great promise as a means for developing detailed age and destination-specific migration flow data from inadequate, partial, and even non-existent information on this most fundamental process underlying population redistribution (Andrei Rogers *et al* 2010) [26].
To describe the migration structure of population the history of migration and the spatial interaction between both origin and destination is important other than the size and composition of the population.
The age patterns of migration have been very well defined and estimated by Coale and Demeny in their regression approach of model life table in 1966. Other remarkable work included Logit system of Brass by Brass (1971), the double exponential graduation of Coale, McNeil, and Trussell (Coale 1977, Coale and McNeil 1972, Coale and Trussell 1974) and a multi-exponential model or Rogers-Castro Model by Rogers and Castro, 1981,1986; Rogers and Watkins, 1987; Rogers and Little, 1994; Rogers and Raymer, 1999; Raymer and Rogers, 2008 [21, 24].
Gravity models, entropy models are popularly called as model spatial distribution of migration. In past decade log linear models have been used to describe the spatial interactions among the populations. Relations among these models have been discussed by Willekens (1980, 1982a, 1982b, 1983) [30, 32, 33]. Multiregional population projection need detailed data sets in absence of which indirect estimation is required (Rogers, 1975, 1985, 1995) [18, 20, 23].
A model based indirect estimation of migration is based on the methodology that concentrates on the analysis of the data with missing values. For this purpose, both age and spatial patterns of migration can be studied to improve the accuracy of the estimates. Generalized log linear models describe the regularities in spatial interactions that can estimate the migration flows with partially available or unavailable data indirectly. Model schedules and generalized models both can be used to smoothen the inadequacies in the data. If the data available is unreliable or then methods imposing regularities from other population is used to estimate the migration flow. When the marginal distribution of matrix of spatial migration is available then spatial patterns of some auxiliary data set such as, recent census or survey, may be utilized as an alternative for the migration pattern to be estimated.

Modelling Spatial Structure of Migration

Migration spatial structure is defined as the arrangement of migration flows in the form of a matrix such that the spatial structure matrix exhibits the level of migration flow when it is compared to a matrix constructed under an assumption of

no interaction effect. The larger the deviations in the migration flow matrix the stronger will be the level of spatial interaction.
A spatial structure matrix is created such that:

1. The matrix of migration flows can be regenerated from a set of parameters.
2. The relative “push” and “pull” factors at origin and destination respectively can be identified.

Generalized linear models can be proved very beneficial in modeling of spatial structure of migration. It offers a mathematical model of migration flow that is readily inferential. The parameters of generalized log linear models can be used to gauge the level of association between any two pair of geographical regions. The models can be applied to multiregional data with one set of parameters displaying the marginal effect of origin, one showing the marginal effects of destination and another set of parameters depicting the combined effect of origin and destinations.

Two Variables Saturated Multipicative Component Model or Log Linear Model

Let O_i be i^{th} the region of origin of migration and D_j be the j^{th} region of destination where both origin and destination are assumed to be independent of each other then the outcome variable, n_{ij} will be the inter regional flow of migration. The multiplicative component model is the saturated model of the log- linear model. A saturated model is a way of data representation in which the components are not constrained. If n_{ij} is the flow observed from region of origin ‘i’ to region of destination ‘j’ then the model is given as-

$$n_{ij} = (T)(O_i)(D_j)(OD_{ij}) \qquad (1)$$

Where T, O_i, D_j and OD_{ij} are the effect parameters? T gives the total effect, O_i denotes the effect of origin and D_j denotes the effect of destination. Therefore, any flow from origin ‘i’ to destination ‘j’ can be expressed by an equation of the form (1) with the corresponding parameters. The parameters when taken together represent the spatial structure of migration (Rogers, Willekens, and Little *et al*. 2002, 2010) [26, 28]. The parameters of the multiplicative component model can be estimated by two techniques namely, (1) Geometric mean effect coding, and (2) Total sum reference coding.

1. Geometric Mean Effect Coding

The overall effect, T, is termed as the constant of proportionality or the size main effect (Willekens, 1983) [29]. It is the geometric mean of all the inter-regional flows values.
The main effect of any region of origin ‘i’ is the ratio of the geometric mean of the flows whose origin is ‘i’ to the overall geometric mean.
The main effect, O_i, shows the relative importance of region ‘i’ as a source of migration (Alonso 1986).
Similarly, the main effect of any region of destination ‘j’ is the ratio of geometric mean of the column ‘j’ to the total geometric mean.
Again, the main effect D_j shows the relative importance of region j over any other region of destination.
Every row (or column) effect is the geometric mean of the row (or column) elements divided by the overall geometric mean, and they are equivalent to the balancing factors of the gravity models (Willekens 1983) [29]. Therefore, they can be compared across regions at different time periods. The matrix, so formed, by the multiplicative components is referred as spatial interaction matrices. OD_{ij} is the interaction effect in equation (1) and each interaction effect is equal to the ratio of the observed migration flow between region ‘i’ to region ‘j’ to the expected flow. i.e.

$$OD_{ij} = \frac{n_{ij}}{(T)(O_i)(D_j)} \qquad (2)$$

The interaction effect OD_{ij} have been inferred as indicators of the accessibility, the ease of communication, or the attraction between the two regions (Rogers, Willekens, Little *et al*. 2002) [28]. It states the departure of the observed flow, n_{ij}, from the expected flow based on the assumption that there is no association between the place of origin and destination. Therefore, the value equal to 1 indicates that there is no association between the origin and destination. A distance from 1 in either direction indicates that there is association between place of origin and place of destination. The values greater than 1 depicts higher accessibility and the values less than 1 depicts lower accessibility of attractiveness between two regions. If the estimated value of interaction effect is equal to 1 it implies that, n_{ij} is determined by the values of $(T), (O_i)$ and (D_j) alone.
The first set of constraints forces the product of the origin (and destination) main effects to be equal to 1. Similarly the product of interaction effect of each row element and column element is equal to 1.
In general, if there are m regions, then there are m^2 linearly independent parameters and 1+m+m+ (mxm) multiplicative components.

1.1 Total Sum Reference Coding

Let T be the total number migrants, (O_i) be the proportion of migrants leaving region ‘i’ and (D_j) be the proportion of migrants entering the region ‘j’ and OD_{ij} be interaction component, i.e. the ration of the observed number of migrants to the expected number of migrants. Then the parameters of the model can be stated as a multiplicative component model using total sum reference coding. Or, we can write,

$$n_{ij} = n_{..}\left(\frac{n_{i.}}{n_{..}}\right)\left(\frac{n_{i.}}{n_{..}}\right)\left[\frac{n_{ij}}{n_{..}\left(\frac{n_{i.}}{n_{..}}\right)\left(\frac{n_{i.}}{n_{..}}\right)}\right] \qquad (3)$$

Where $n_{..}$ is the total number of migrants in the given time period. O_i is the total number of migrants from the region of origin *'i'* and D_j id total number of migrants to the place of destination *'j'*.

The origin component denotes the share of migrants from each region, 'i', the destination component indicates the total share of migration to each region, 'j'. The relationship among the parameters is expressed by the constraints (Raymer, Bonaguidi and Valentini 2006) [13].

2. Two Variable Log Linear Additive Model

The log-linear additive model is the linear function of the logarithms of the multiplicative component model (Knoke and Burke 1980) [7]. The model is given as-

$$\ln(n_{ij}) = \ln(T) + \ln(O_i) + \ln(D_j) + \ln(OD_{ij})$$

Or,

$$\ln(n_{ij}) = \lambda + \lambda_i^O + \lambda_j^D + \lambda_{ij}^{OD} \qquad (4)$$

The λ values represent the natural logarithms of the components in equation (1). The interaction parameters, λ_{ij}^{OD}, are known as the log odd ratios i.e. the ratio of logarithm of the two odds, the odds of migration from region of origin 'i to the destination 'j' rather than migration to reference region and the odds of migration from the reference region to the destination 'j' rather than migration to the reference region itself. It therefore indicates the relative likelihood of one outcome to another. The value of odds ratio equals to one shows the independence of the two parameters or in this case independence of two regions. A positive value of the odds ratio represents positive association and vice versa.

Therefore, we can write,

$$\lambda_{ij}^{OD} = \ln\left[\frac{n_{ij}/n_{im}}{n_{mj}/n_{mm}}\right] \qquad (5)$$

There are mxm linearly independent parameters in a multiregional migration system with m regions. Therefore, the set of parameters, *1, m, m and (mxm)* for Total, Origin, Destination and interaction effect respectively are estimated by contrast coding scheme so that both independent and dependent parameters are estimated. The multiplicative component gives the estimated value for *1+ m+ m+ (mxm)* parameters though they are not linearly independent of each other, therefore resulting in one value for total, (m-1) value for origin, (m-1) values for destination and (m-1) x(m-1) values for interaction effects. Contrast coding scheme fixes all linear additive parameters for a region equal to 0 except that particular region. Therefore, the model estimates (m-1) x(m-1)-1 values for the parameters. For example, in the current study, it fixes the parameters for the last region, $\lambda_m^O = \lambda_m^D = \lambda_{mj}^{OD} = \lambda_{im}^{OD} = 0$

Numerical Illustration and Discussion

To illustrate the described models in the paper, the data has been taken from Census of India, 2011, migration tables and divided into five major regions according to their geographical position in the map of India.

North: Jammu and Kashmir, Himachal Pradesh, Punjab, Chandigarh, Uttarakhand, Haryana, Delhi, Rajasthan and Uttar Pradesh.

East: Bihar, Jharkhand, Odisha, West Bengal, Andaman and Nicobar Islands.

West: Gujarat, Maharashtra, Madhya Pradesh, Chhattisgarh, Goa, Daman and Diu and Dadra and Nagar Haveli.

South: Andhra Pradesh (Telangana included), Karnataka, Kerala, Tamil Nadu, Lakshadweep and Puducherry.

Northeast: Arunachal Pradesh, Assam, Manipur, Meghalaya, Mizoram, Nagaland, Sikkim and Tripura.

Table 1 represents inter-regional migration flow in India, 2001. This is the Multiregional Migration flow matrix calculated from the Migration tables of Census of India, 2001, which is the basic matrix used for explanation of the migration models.

Table 1: Inter-regional migration in India, 2001

Origin	Destination					
	North (1)	East (2)	West (3)	South (4)	Northeast (5)	Total (O_i)
North (1)	30272421	289203	3078830	258721	41012	33940187
East (2)	1908825	21564793	1423624	413669	143819	25454730
West (3)	859082	144111	34353720	497668	7452	35862033
South (4)	159883	91486	958048	33876710	9286	35095413
NorthEast (5)	83560	174294	47326	51694	1175457	1532331
Total (D_j)	33283771	22263887	39861548	35098462	1377026	131884694

1. Multiplicative Component Model using Geometric Effect Coding

The two variable saturated log-linear model or multiplicative model is written as equation no. (1). Using table 1 and the equation (2) and (3) the components of the migration flow matrix also called as spatial interaction matrices are estimated, shown in Table 2.

Table 2: Spatial Interaction Matrix using Geometric Effect Coding for 2011

Origin	Destination					
	North	East	West	South	Northeast	Total (Oi)
North	19.23	0.396	1.227	0.243	0.441	1.709
East	0.735	17.887	0.344	0.236	0.937	2.818
West	0.974	0.352	24.43	0.835	0.143	0.957
South	0.234	0.288	0.879	73.38	0.230	0.742
Northeast	0.310	1.394	0.110	0.284	73.810	0.292
Total (D_j)	2.022	0.939	3.224	1.366	0.120	4,55,532.07

The overall effect or the size main effect T is calculated as 363546.333. The main effect O_i shows the relative importance of the place of origin with respect to any other place of origin.
E.g. we estimate the main effect of region of origin 5 equals to 0.296
O_5= 0.296 is the least among all the estimated main effects of origins. This implies that among the five regions, the fifth region i.e. the Northeast region of India is the least important place of origin. Therefore, it can be said that the people of Northeastern region move the least to other regions.
Similarly, the main effect of destination, D_i, is estimated which shows the relative attractions in the place of destination. E.g the main effect of the place of destination, 5, Northeast region, is calculated as 0.158
The value of D_5 = 0.158 is the least among all the main effects D_is, this shows that the 5th place of destination is the least attractive place of destination among all. In the similar pattern, it is observed that among the five regions, the 2nd place of origin (Easter region) and the 4th place of destination (Western region) are the major source of migration and most attractive destinations respectively.
Each interaction effect OD_{ij} represents the accessibility or the attractiveness between the two regions, 'i' and 'j' (Rogers, Willekens, Little *et al*. 2002) [28]. The value of OD_{ij} equal to unity shows that the migration between the place of origin and destination doesn't depend upon each other. In the present study, we have observed that most for the rows and columns are not independent of each other, specially the interactions at the diagonal. Six inter-regional flows namely, north to West, East to Northeast, South to West, West to South, West to Northeast and Northeast to east have the value of OD_{ij} close to unity, which implies that the migration in these regions is independent of the other regions. This implies that some parameter values, which are not independent with each other, can be estimated from others. There is one total effect, 10 main effects and 25 main effects, total 36 estimated from 25 observed flows making 11 others redundant. The constraints expressed in the models show the relationship between main effects and the interaction effects respectively. If four of the interaction effects are given than the fifth can be explained using other components.

2. Multiplicative Component Model using Total Sum Reference Coding

Table 2 explains the components of the spatial interaction matrix, has been derived using the observed inter-regional migration flow from the table 1 and the total sum reference coding explained in section 1(b).

Table 3: Multiplicative Components using Total Sum Reference Coding

Origin	Destination					
	North (1)	East (2)	South (3)	West (4)	Northeast (5)	O_i
North (1)	3.410	0.074	0.032	0.272	0.067	0.264
East (2)	0.310	4.806	0.042	0.151	0.237	0.200
South (3)	0.031	0.025	4.418	0.155	0.019	0.219
West (4)	0.097	0.030	0.055	2.963	0.008	0.291
Northeast	0.162	0.275	0.030	0.038	34.773	0.027
D_j	0.261	0.178	0.212	0.323	0.026	120660490

The column 'O_i' and row 'D_i' explains the proportion of migrants from ith origin and the proportion of migrants to the jth destination in total number of migrants '(T)' respectively. It is observed that 26 percent of overall migrants originate from the northern region of the country. This percentage is minimum from the northeast region i.e. 0.02 percent. Similarly 32 percent of the total migrants is attracted to the Western region of the country which the highest among all destinations. This percentage is least for Northeast region, which shows that the Northeast region of the country shows the minimum participation in inter-regional migration across the country. Using the expression for estimate of interaction parameters 'n_{ij}' from models 2 the population of inter-regional migrants can be predicted, that is shown in the table 3.

Table 4: Predicted Migration flows under Multiplicative Component Model using Total Sum Reference Coding

Origin	Destination					
	North (1)	East (2)	South (3)	West (4)	Northeast (5)	O_i
North	28318772	420116	217135	2793803	54433	31804259
East	1948071	20596863	212657	1175037	146381	24079008
South	212867	115113	24738884	1318316	12840	26398019
West	894878	185196	407536	33662556	7296	35157463
Northeast	136160	157816	20383	39078	2868303	3221741
Total	31510747	21475104	25596595	38988790	3089253	120660490

3. Log Linear Model

The log linear model is expressed in equation 5. The parameters are estimated using Generalized log-linear Poisson Model explained in Appendix (1). The predicted spatial structure matrix is given in Table 4.

Table 5: Estimates of Additive linear parameters using "last region" coding

Origin	Destination					
	North (1)	East (2)	South (3)	West (4)	Northeast (5)	O_i
North	9.302	4.944	6.33	8.234	0	-3.965
East	5.636	7.847	5.32	6.379	0	-2.975

South	5.856	5.093	12.51	8.927	0	-5.409
West	7.857	6.134	8.969	12.733	0	-5.974
Northeast	0	0	0	0	0	0
D_j	-3.048	-2.9	-4.947	-4.296	0	14.623

The association or interaction parameters in table 5, λ_{ij}^{OD}, are the logged odd ratios, equation (10). λ_{23}^{OD} Indicates the logged ratio of migration to the region 3 rather than region 5, between a migrant originating in region 2 and region 5. Or, the odds of a migrant from region 2 to move to region 3 over region 5 is 5 times higher than a migrant from region 5 to move to region 3 over region 5. The logged odds ratio equals to zero means complete statistical dependence. The parameter estimates in Table 5 is in logarithmic form so to simplify, they may be converted to the multiplicative form by taking exponentiation. The additive linear components of parameters for the last region, place of origin/destination are equal to zero and therefore they make no contribution in the equation (9).

Conclusion

The saturated model employed in this study provides a simplified yet comprehensive explanation of the spatial structure of inter-regional migration when compared to other log-linear formulations. The Multiplicative Component Model with geometric mean effect coding uses the geometric mean as the reference category and estimates $1 + m + m + (m \times m)$ parameters, which are mutually dependent. Although both the geometric mean and total sum reference saturated models satisfy the underlying structural equation, the total sum reference model offers clearer empirical interpretation. The parameters derived from both linear additive and multiplicative saturated log-linear models successfully capture the spatial migration structure, allowing full regeneration of interaction effects. The multiplicative component model is particularly relevant as it is formally equivalent to the gravity model (Willekens, 1983) [29].

Regional analysis reveals that the North-Eastern region contributes the lowest share of inter-regional migrants, despite substantial intra-regional mobility, likely influenced by geographical isolation and limited transport infrastructure. The Eastern region emerges as a major source of out-migration, accounting for nearly 23 percent of total inter-regional outflows. In contrast, the Western region records the highest share of in-migration, driven by major metropolitan centres such as Mumbai and the presence of large industrialized states offering extensive employment opportunities and superior transport and communication facilities. The Northern region also exhibits significant in-migration. Furthermore, migration flows between the northern and southern regions are not independent, indicating the presence of multiple socio-economic and structural factors shaping inter-regional migration patterns in India.

Appendix I

Generalized Poisson Log Linear Model

The Generalized linear model is constructed with three components that explain the model. The random component or the response variable, 'Y', the explanatory variable or systemic component and 'link' which explains the functional relationship between the mean (expected value) of the response variable and the explanatory variable. Therefore, under GLM, the relationship between the explanatory variable and a function of mean is expressed in a linear form through which the response variable can be estimated.

Let there be a random sample $(Y_1, Y_2, \dots Y_k)$ of size k with independent observations, Y_i being migration count from one region to another, and μ be the expected value of Y, i.e. $E(Y) = \mu$. Let 'X' be an explanatory variable defined as the migration counts of the origin and destinations. Each observation in a migration study is the number of individuals migrating from one place to another, which is discrete in nature. Since, each response observation is a non-negative count, we can assume that the response variable 'Y' follows Poisson distribution. The value of μ varies in accordance with the level of the explanatory variables. The explanatory variable 'X', intends the role of $\{x_j\}$ in the model. Some $\{x_j\}$ can be stated as the linear combination of the $x_j's$ called as linear predictor or can be based on other model to consider the interaction effect in their effects on 'Y', i.e. $x_3 = x_1 x_2$. The third GLM component is *link*. Link is defined as the functional relationship between response and explanatory variable. The model formula for link function is -

$$g(\mu) = \alpha + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_k x_k$$

Where, $\alpha \; and \; \beta_1, \beta_2, \dots \beta_k$ are constants and the function g (.) is called as link function. In the present GLM, the mean cannot be negative, so the link function $g(\mu) = \log(\mu)$ is applied. And therefore, the model here is called Generalized Poisson loglinear model. The log mean is the parameter of the Poisson distribution and the log link is canonical link for a Generalized Log Linear Model since it uses the natural parameter $g(\mu)$ with Poisson random variable.

The Poisson log linear model is written as-

$$\log \mu = \alpha + \beta x$$

One unit change in X lays a compound effect of e^{β} on μ. The mean of Y at x+1 equals to mean of Y at x multiplied by e^{β}.

If $\beta > 0$, then $e^{\beta} > 1$, the mean of Y increases as X increases. If $\beta < 0$, then the mean of Y decreases as X increases. And when $\beta = 0$, implies that $e^{\beta} = 1$, implies that the mean of Y doesn't change with a change in X.

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