



Multi-vendor multi-buyer supply chain optimization model with perishable products

Binay Kumar

Binay Institute of Training and Research, Delhi, India

Abstract

Supply Chain Management (SCM) is a set of approaches tied up to integrate various stake holders along the supply chain namely, suppliers, manufacturers, warehouses, distributors, retailers, customers and all others who are adding value in fulfilling the customer requirements. SCM has gained enormous amount of importance among the participating companies and have realized it as an inevitable mode to achieve competitive advantage. In this paper we have developed mixed integer programming model for multi-vendor multi buyer supply chain problem with perishable products. The objective is to determine a periodic policy of production, supply and transportation that minimizing the sum of the inventory and transportation costs. A numerical case study is elucidated to demonstrate the salient features of the proposed model.

Keywords: vendor, buyer, supply chain, perishable products, mixed integer programming

1. Introduction

Supply chain management is becoming an essential component of effective decision making for organization. Now a days increasing global competition involved that products must be produced and supplied in the right place at the right time at a low cost. We consider perishable products. Many items in practice are perishable. Fruit, vegetables, meat, foodstuffs, perfumes, alcohol, gasoline, photographic films, etc., are just some examples where deterioration may typically occur during their normal storage period. If the deterioration rate is significant, its impact on the inventory-transportation system cannot be ignored. Perishability refers to the possible physical deterioration of a product and is different from its obsolescence, which generally refers to cases where the demand for the product becomes negligible after a certain time. Perishable products can be subdivided into two categories, according to the characteristic associated with their life span:

1. Products with fixed life span, when products maintain their usefulness for a fixed period of time and then must be deemed useless;
2. Products with a variable life span, if their usefulness diminishes over time, according to the product age.

In the present paper, we develop a mathematical model of multi-vendor multi buyer model for perishable products. Here we consider production rate of all the vendor is equal to demand rate of all the buyers. We extent the problem considered by Zanoni and Zavanella (2007) ^[10]. They assumed single-vendor single-buyer with integrated transport-inventory system in the case of perishable products.

The detailed description of the paper is as follows: Section 2 review the current literature on integrated transport-inventory Supply chain optimization problems. Section 3 describes the problem formulation with notation and assumptions. Section 4 presents Formulation of Mathematical model. In Section 5, the proposed model is illustrated by a numerical example and solve the numerical example by LINGO in section 6. Finally, conclusions are discussed in Section 7.

2. Literature Review

The first work on perishability is presented by Ghare and Schrader (1963) ^[9]. Fries (1975) considered optimal order policies for a perishable commodity with fixed life time. Nahmias (1982) pointed out that inventory decay could exert a significant impact on the overall inventory costs and presented comprehensive review on perishable products. Goyal and Gupta (1989) developed the buyer vendor coordination inventory models. Nandakumar and Morton (1993) studied near myopic heuristic for the fixed life perishability problem. Liu and Shi (1999) developed (s, S) model for inventory with exponential lifetimes and renewal demands. Liu and Lian (1999) developed (s, S) model for inventory with fixed life time. Lian and Liu (2001) studied continuous review perishable inventory systems. Goyal and Giri (2001) proposed review of the classification of perishable products and policies for managing them. Sphicas (2006) analyzed EOQ and EPQ with linear and fixed backorder costs. Wong *et al.* (2009) developed coordinating supply chains with sales rebate contracts and vendor-managed Inventory. Yongrui Duan *et al.* (2010) presented buyer vendor inventory coordination with quantity discount for fixed lifetime product. Cardenas-Barron (2011) developed the derivation of EOQ/EPQ inventory models with two backorder costs using analytic geometry and algebra. Hung-Chi Chang (2011) ^[5]

analyzed a comprehensive note on an economic order quantity with imperfect quality and quantity discounts.

3. Mathematical Formulation

We consider the problem where a set $I = \{1, \dots, I\}$ of perishable products have to be shipped from multi-vendors to multi buyers. Production rates of each product at vendor's node are equal to demand rates of each product of multi buyers

Assumptions & Notation

We make the following assumptions:

1. Production rate of all the vendor is equal to demand rate of all the buyers.
2. Inventory cost per time unit of each type of product is different at vendor & buyers nodes.
3. Neither setup costs at vendors' node nor order costs at buyers' node are considered since, for each product type, the production rate is equal to demand rate. The absence of set-up cost can be easily justified.
4. Inventory shortages are not permissible over time horizon.
5. Transportation costs are independent from the quantity transported, i.e. we assume a fixed cost for each vehicle and shipment.
6. An unlimited number of vehicles with given capacity is available.
7. Products are perishable: they have to be produced, transported and sold by an expiry date which is different for each product type. This date may represent either the real expiry date or a deadline, nearer than the expiry date, by which the product type is to be sold in order to guarantee the final customer a product with a sufficient residual life.
8. products are assumed to be processed in each node according to a FIFO(First In First Out)

Parameter

a	index for vendors (a=1,2,3.....n)
b	index for buyers (b=1,2,3.....,n)
i	index for product (i=1,2,3.....,I)
j	index for shipment frequency (j=1, 2,3.....,J)
t	index for discrete time instant (t=0,1.....t.....,H-1) & $t \in T$
k	index for shipment time instant (k=1,2,3.....,K)

Notations

H	Time horizon
V_a	Set of multi vendors $\{a_1, a_2, a_3, a_4, \dots, a_n\}$
B_b	Set of multi buyers $\{b_1, b_2, b_3, b_4, \dots, b_n\}$
T	Set of discrete time instants $\{0, 1, \dots, t, \dots, H-1\}$
I	Set of product indices $\{0, 1, \dots, t, \dots, H-1\}$
J	Set of frequency at which it is possible to carry out a shipment
K	Set of all shipment time instants
f_j	j th shipment frequency ($j \in J$)
t_j	Period corresponding to frequency f_j i.e. $\left(t_j = \frac{1}{f_j}\right)$
M_{ia}	Production rate of product i at the each vendor.
M_{ib}	Demand rate of product i at the each buyer.
v_i	Volume per unit of product i
$d_i^{V_a}$	Initial inventory of product i at vendor node
$d_i^{B_b}$	Initial inventory of product i at buyers node
S_{iabk}	Quantity of product i shipped at time t from the vendor to the buyers
y_j	Number of vehicles used at frequency f_j
r	Transportation capacity of each vehicles
c	Transportation cost per journey performed
$I_{it}^{V_a}$	Inventory of product i at vendor node

- $I_{it}^{B_b}$ Inventory of product i at buyers node
- $\bar{I}_i^{V_a}$ Average inventory of product i at vendor node
- $\bar{I}_i^{B_b}$ Average inventory of product i at buyers node
- $h_i^{V_a}$ Inventory cost of one unit of product i per time unit in vendor node
- $h_i^{B_b}$ Inventory cost of one unit of product i per time unit in buyer's node
- X_{ij} Fraction of product i shipped at frequency f_j
- $K_t = \{k \in K : k \leq t\}$ Set of shipping time instants lower than or equal to t .

4. Formulation of mathematical model

The instantaneous value of the inventory for the product i in vendor's node is equal to the initial value of the inventory at the vendor's node minus the cumulated quantity sent to multi buyer up to time t plus the quantity produced.

$$\sum_{a=1}^n I_{it}^{V_a} = \sum_a d_{it}^{V_a} + \sum_{a=1}^n M_{ia} t - \sum_a \sum_b \sum_k S_{iabk} \quad \forall i, t \tag{1}$$

The instantaneous value of the inventory for product i at buyers node is equals to the initial value of the inventory at the buyer node plus the cumulated received by vendor node up to time t minus the quantity demanded

$$\sum_b I_{it}^{B_b} = \sum_b d_{it}^{B_b} - \sum_b M_{ib} t + \sum_a \sum_b \sum_k S_{ibk} \quad \forall i, t \tag{2}$$

Average inventory in vendor node is equal to

$$\sum_{a=1}^n \bar{I}_{it}^{V_a} = \sum_a d_{it}^{V_a} + \sum_a M_{ia} \cdot \frac{H}{2} - \frac{1}{H} \sum_a \sum_b \sum_t (H - t) S_{iabt} \quad \forall i, t \tag{3}$$

Average inventory in buyer's node is equal to

$$\sum \bar{I}_{it}^{B_b} = \sum_b d_{it}^{B_b} - \sum_b M_{ib} \cdot \frac{H}{2} + \frac{1}{H} \sum_a \sum_b \sum_{t \in K} (H - t) S_{iabt} \quad \forall i, t \tag{4}$$

Total sum of transportation and inventory costs is given by

$$TC = \sum_a \sum_b \sum_i \sum_j \frac{\sum_a h_i^{V_a} + \sum_b h_i^{B_b}}{2} t_j \cdot M_{ib} \cdot X_{ij} + \sum_{j \in J} \frac{c}{t_j} \cdot y_j \tag{5}$$

Objective function define as to minimize total inventory & transportation costs

$$\text{Min} \left\{ \sum_a \sum_b \sum_i \sum_j \frac{\sum_a h_i^{V_a} + \sum_b h_i^{B_b}}{2} t_j M_{ib} \cdot X_{ij} + \sum_{j \in J} \frac{c}{t_j} \cdot y_j \right\} \tag{6}$$

Demand Constraints

Define as for each product $i \in I$ the total quantity shipped in a cycle has to be equal to the quantity produced in vendor node and absorbed (sold) in vendors node.

$$\sum_{j \in J} x_{ij} = 1, \quad i \in I \tag{7}$$

Capacity Constraints

Quantity shipped at jth frequency should be less than the transportation capacity of jth shipment.

$$t_j \sum_{i \in I} v_i \cdot M_{ia} \cdot x_{ij} \leq r \cdot y_j, \quad j \in J, \tag{8}$$

Perishability Constraints

It is necessary that production, transportation and sale occur within the corresponding expiry date ts_i (for each product $i \in I$)

$$\sum_{j \in J} t_j \cdot x_{ij} \leq ts_i \quad i \in I, t \in T \tag{9}$$

Decision variables are

$$x_{ij} \geq 0 \quad i \in I, \quad j \in J, \tag{10}$$

$$y_j \geq 0 \text{ and integer} \quad j \in J \tag{11}$$

Since products are assumed to be processed in each node following a FIFO, the perishability constraints to be satisfied are

$$\sum_a I_{it}^{Va} + \sum_b I_{it}^{Bb} \leq M_{ia} \cdot ts_i \quad \forall i \in I, t \in T \tag{12}$$

In this way, total amount of product i , available in the system at every time t , to be less than or equal to value $M_i \cdot ts_i$. It can be observed that the total inventory of each product in the two nodes is constant over time (Bertazzi & Speranza 1999a)

$$\sum_a I_{it}^{Va} + \sum_b I_{it}^{Bb} = \sum_a d_i^{Va} + \sum_b d_i^{Bb} \quad \forall i \in I, t \in T \tag{13}$$

Then constraints (12) become

$$\sum_a d_i^{Va} + \sum_b d_i^{Bb} \leq M_i \cdot ts_i \tag{14}$$

5. Numerical Example

A leading dairy company in India distributes milk and converts the product like butter, cheese, ghee etc. and packages it with branded name through distributor sends to retailer. The dairy company in a specific city has three production units where the company produces brand product in different size packet with standard weight. Then the product is transported to three distribution centers and then retailers receive products from these through which the products reaches to customers. From wastage and limits of storage capacity the company decides to control the production rate comparable to demand so that an optimum demand and supply is maintained and the company is not looser either way. Company makes a plan for one year for inventory distribution and transportation policy. Over a year distribution center's shipped of products in four times as 2nd month, 5th month, 8th month and 11th month.

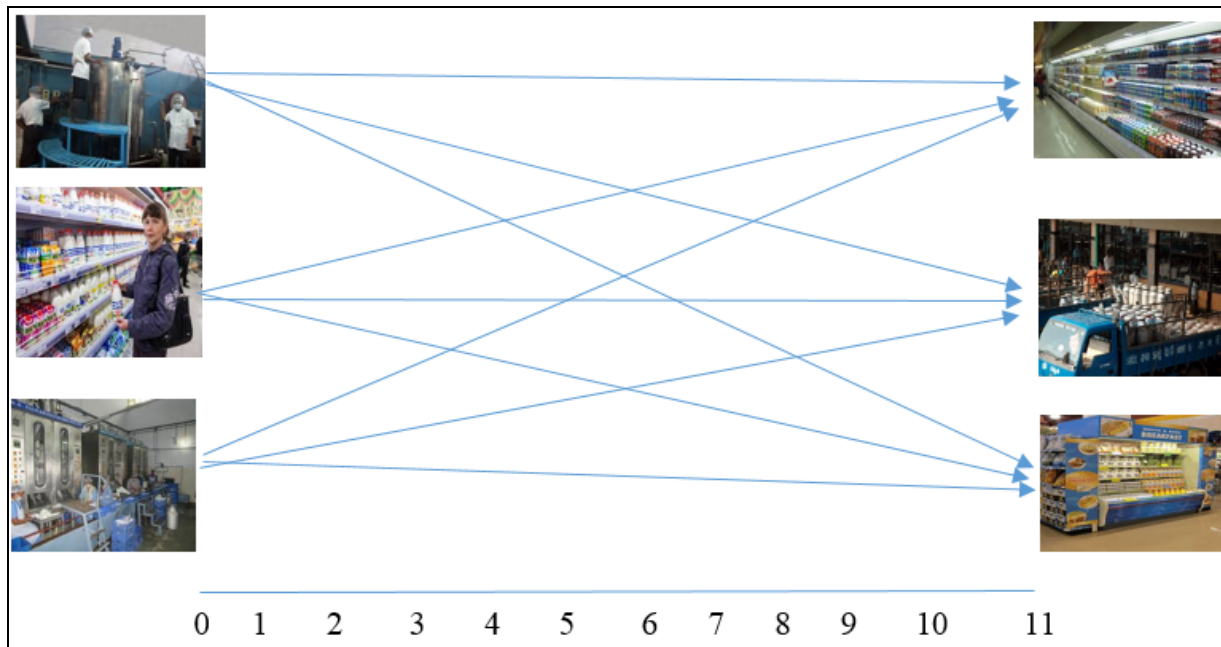


Fig 1: Three production centers & three distribution centers

These milk converted products are considered as perishable product. Each product has a time period, after that period products have no importance because over the time period quality of the products get reduces and totally considered as wastage. So it is necessary to make optimal inventory decision for each frequency when demand occurs in three distribution centers. Production unit center and each distribution center have some storage capacity, so inventory holding cost is different and distribution center have more holding cost than production center. Here decision has to taken for optimal inventory shipped in each frequency and number of truck used in each frequency shipment. Each truck has same capacity and capacity measure in term of volume. The following information is given as:

Transportation capacity of each truck is 2000, transportation cost per shipment is R.s 10,000.00, Production rate of Butter, Cheese & Milk powder are 200, 225 & 250. Production rates of three distribution centers is equal to demand rates of three retailers. Perishable time of these three products is 4 months. Volume of the Butter, Cheese & Milk powder of each pack are 10, 12 & 11.

Table 1: Quantity of three products shipped at different frequencies

	1 st Product (Butter)			2 nd product (cheese)			3 rd product (Milk Powder)		
	B1	B2	B3	B1	B2	B3	B1	B2	B3
f_1	$s_{111}=11$	$s_{121}=8$	$s_{131}=6$	$s_{211}=12$	$s_{221}=14$	$s_{231}=8$	$s_{311}=12$	$s_{321}=9$	$s_{331}=13$
f_2	$s_{112}=8$	$s_{122}=7$	$s_{132}=6$	$s_{212}=11$	$s_{222}=10$	$s_{232}=8$	$s_{312}=13$	$s_{322}=12$	$s_{332}=8$
f_3	$s_{113}=8$	$s_{123}=7$	$s_{133}=6$	$s_{213}=11$	$s_{223}=10$	$s_{233}=8$	$s_{313}=13$	$s_{323}=12$	$s_{333}=8$
f_4	$s_{114}=8$	$s_{124}=7$	$s_{134}=6$	$s_{214}=11$	$s_{224}=10$	$s_{234}=8$	$s_{314}=13$	$s_{324}=12$	$s_{334}=8$

Table 2: Initial inventory of vendor & buyers

Initial inventory	V1	V2	V3	B1	B2	B3
Butter	13	10	15	10	7	10
Cheese	15	16	12	8	9	6
Milk Powder	12	18	16	9	11	5

Table 3: Inventory cost of vendor & buyers

Inventory cost	V1	V2	V3	B1	B2	B3
Butter	2			3	4	5
Cheese	1			2	3	4
Milk Powder	5			4	3	2

Table 4: Production rate of product at the vendor

Product	Butter	Cheese	Milk Powder
Production Rate	200	225	250

Table 5: Expiry time of Perishable product

Product	Butter	Cheese	Milk Powder
Expiry time (in Months)	4	5	4

6. Solution of numerical example

LINGO 11.0 version software has been used to provide numerical solution of this case study. LINGO is an effective tool designed to make building and solving mathematical optimization models easier and more efficient. When developing a model in LINGO, it helps to understand how the model is processed internally by the LINGO solver. The relationships in your model influence the computation time, the solution methods used by LINGO and the type of answer returned. Here we'll explain some of the different types of relationships in a LINGO model and how each type affects the solution search. An understanding of these topics is not required to use LINGO but it can help you use the software more effectively. Local optimal solution found using LINGO 11.0 & Total minimum Transportation & inventory cost is Rs 80000.00.

7. Conclusion

This paper presents the ways to minimize the cost of distribution; storage and transportation in the competitive scenario which can help in optimize the profit and loss vis-a-vis the initial expenditure and facilitate decision makers to develop effective decision policy. The market driven economy requires a full proof mechanism to ensure the sustainable growth. The infra structure development and facilities provided to work force look forward a better economy and therefore the distribution, storage & transportation is a parameter that has to be addressed for better application. In this paper I need to develop better inventory distribution & transportation policy for vendor to buyers to maintain the balance in production & distribution. In future research, different extensions to the proposed model can be considered for multi-vendors to multi buyers.

8. References

- Goyal SK. An integrated inventory model for a single supplier-single customer problem. *International Journal of Production Research*. 1976; 15(1):107-111.
- Hill RM. The single-manufacturer single retailer integrated production-inventory model with a generalized policy. *Eur J Oper Res*. 1997; 97(3):493-499.
- Lal R, Staelin R. An Approach for Developing an Optimal Discount Pricing Policy, *Management Science*, 30, 12, 1524-1539.
- Joglekar PN, Tharthare S. The individually responsible and rational decision approach to economic lot size for one vendor and many purchasers. *Decision Sciences*. 1990; 21:492-506.
- Banerjee A, Banerjee S. A coordinated order-up-to inventory control policy for a single supplier and multiple buyers using electronic data interchange. *Int J Prod Econ*. 1994; 35(1-3):85-91.
- Lu L. A one-vendor multi-buyer integrated inventory model. *European Journal of Operational Research*. 1995; 81:312-323.
- Viswanathan S, Piplani R. Coordinating Supply Chain Inventories through Common Replenishment Epochs, *European Journal of Operational Research*. 2001; 129:277-286.
- Woo YY, Hsu SL, Wu S. An integrated inventory model for a single vendor and multiple buyers with ordering cost reduction. *International Journal of Production Economics*. 2001; 73(3):203-215.
- Ghare PM, Schrader SF. A model for exponentially decaying inventory. *Journal of Industrial Engineering*. 1963; 14:238-243.
- Zanoni, Lucio Zavanella. Single-vendor single-buyer with integrated transport-inventory system: Models and heuristics in the case of perishable goods, *Computers & Industrial Engineering*. 2007; 52(1):107-123.
- Bertazzi L, Speranza MG, Ukovich W. Exact and heuristic solution for a shipment problem with given frequencies". *Management Science*. 2000; 46:973-988.
- Bertazzi L, Speranza MG. Inventory control on sequences of links with given transportation frequencies". *International Journal of Production Economics*. 1999; 59:261-270.