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Mathematical modeling and simulation of quarter car model of an active air suspension

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Abstract
Automobile suspension have the function of improving passenger comfort by isolating vehicle from external disturbances and increase holding ability of the vehicle by providing adequate suspension deflections. This work presents an improved model of an active control strategy presented in an earlier work. The work presented an active control system for an air suspension based on mass flow control. In the earlier model of a single degree of freedom (SDOF), system had been considered. The present work gives a more refined version of earlier model of quarter active suspension. Two degree of freedom quarter car model is considered. Three different control strategies of an active control are modeled and simulated. It is seen that an active control system with the mass flow rate of air as a function of relative velocity of sprung mass to unsprung mass has transmissibility less than 1 for low value of damping and gain equal to 1 in the system. Similar response is seen if the active control system has low damping and gain equal to -1 with mass flow rate of air to airspring as a function of sprung mass velocity. These two strategies are proposed be used in the active air suspension system.

Keywords: active vibration control, airspring, pneumatic suspension, quarter car model

1. Introduction

Vehicle suspension system is basically a system of wheel tires, springs, shock absorbers and linkages that connects a vehicle to its wheels and due to which relative motion between the two is possible. Various types of suspension systems have been developed. It serves a dual purpose, contributing to the vehicle's road holding/handling, braking for good active safety, driving the pleasure and keeping vehicle occupants comfortable. A ride quality reasonably well isolated from road noise, bumps, vibrations, etc. It is important for the suspension to keep the road wheel in contact with the road surface as much as possible because all the road or ground forces acting on the vehicle do so through the contact patches of the tires.

A pneumatic suspension is one such suspension. Air springs in the form of bellows are used in pneumatic suspensions. One of the most prominent advantages of pneumatic suspension over metallic counterparts is the fact that the main natural frequency of the system can be made independent of sprung mass. Most of the commercially available passenger cars are equipped with passive spring and damper suspension systems. In recent years, a few commercially available cars are fitted with active suspension systems.

2. Literature Review

The complex dynamic stiffness of a damped air spring connected to a tank was obtained [1]. The effect of vibration frequency, vibration amplitude, and volume of auxiliary chamber in air springs with the auxiliary chamber was studied [2]. A model was proposed in dimensionless parameters to understand the effect of the parameters, which define the suspension and to select the spring type [3]. A model of the pneumatic suspension, which includes a nonlinear fluid dynamics model, for the suspension stiffness, damping factor and transmissibility, was developed [4]. It is experimentally found that the sizes of the pipe, the tank and the air spring play an important role in determining the suspension's behavior. An airspring with auxiliary chamber connected through an orifice was modeled for calculation of dynamic stiffness of air spring [5].

Despite the widespread use of pneumatic vibration isolation systems, the possibilities of using the pneumatic configuration as an active system have not been sufficiently studied. An active quarter car pneumatic active system where control is exercised by measurement of system states and use of the state information to drive the air pumps to control the pressure in air spring and consequently the spring forces produced was developed [6]. A quarter car model of a pneumatic active car suspension to show the advantages of active control was developed. Control laws were derived using limited state feedback, linear stochastic optimal theory for a quarter car model [7] Active pneumatic spring constructed as flow chamber with changing volume and pressure which is obtained by relative displacement of the piston was considered [8].

Ali M. studied and discussed the comparison between passive and a semi-active system with PID and fuzzy controls is discussed and presented with an aim to achieve optimum ride with the minimum dissipation of energy [9].

V. Bhandari and S. Subramanian [10] developed a control scheme for an electronically controlled pneumatic actuator that can be used in suspensions of commercial vehicles. The performance of the suspension system with regards to different inputs is simulated using the quarter car model and the half car model have been studied theoretically and experimentally [11].

Wang, Zhang and Du presented a novel design for demand dependent active suspension (DDAS) which focuses on vehicle stability using active suspension system. Demonstration of the experimental investigation validating the effectiveness and capability of DDAS in counteracting vehicle rollover moment in carried out in this paper [12].

The research on active air suspension has primarily revolved around some type of adjustable dampers or force actuators that directly impact damping of the suspended mass. The use of airspring as a force actuator limits the mass on the airspring.In the earlier work, a novel strategy of active control of air suspension based on mass flow control had been proposed. In this approach since the airspring is modeled as a spring and not as an actuator. It is concluded that the active suspension using velocity feedback as the control strategy has lower transmissibility and hence better performance at resonance compared to passive suspension [13].

In the earlier work the model considered is a SDOF system. In order get a better insight into the response of the active air suspension, the model is further refined. The model considered in this paper is a two degree of freedom system quarter car model of an active air suspension. The sprung mass is supported by an airspring and damper. The unsprung mass is supported by a tyre.

Active control is achieved by controlling the mass flow rate of air into the air spring. Mass flow rate of air into airspring is made a function of one the following:

Strategy I- Relative velocity of sprung mass and unsprung mass,
Strategy II- Velocity of unsprung mass,
Strategy III- Velocity of sprung mass.

3. Objective

Develop and analyze a two degree of freedom quarter car model of an active air suspension. The model consist of a sprung mass suspended on air spring. The unsprung mass is suspended by tire. To achieve this objective following methodology is proposed.

4. Methodology

A mathematical model of a quarter-car model of an active air suspension is presented. Three different strategies using mass flow control of air to the airspring are presented.

Methodology includes:

- Obtain the equations of motion of the active air suspension.
- Obtain the response equations of the active air suspension.
- Simulate the mathematical model of the equations of motion using MATLAB.
- Propose the most suitable strategy for use in active air suspensions based on mass flow control of air to the airspring.

5. Mathematical Modelling

Fig. 1 shows a quarter car model of an active pneumatic. The system is to be a closed loop control system. Area of the air spring is assumed to change negligibly. The weight exerted by the sprung mass is supported by the air spring. It consists of the sprung mass m_s of the vehicle supported by an air spring in parallel with a damper. The system does not have a separate damper. The damper represents the inherent damping in the suspension. The front air spring has an area A and static pressure inside is P. The damping coefficient at of suspension is C_s. The stiffness of the tire is given by k_t. The bounce motion of the sprung mass is x_s and that of unsprung mass is x_u. The excitation from the road to the base of the wheel is x_r.

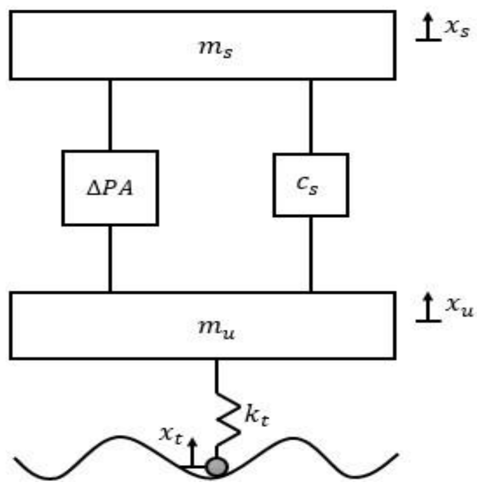

Fig 1: Quarter car model

Equation of motion is

$$m_s + \ddot{x}_s + c_s(\dot{x}_s - \dot{x}_u) - \Delta PA = 0 \quad (1)$$

$$m_u\ddot{x}_u - c_s(\dot{x}_s - \dot{x}_u) + k_t(x_u - x_r) + \Delta PA = 0 \quad (2)$$

Relation between rate of change of pressure in the airspring $\dot{P}$ and the mass flow rate of air into airspring $\dot{m}$ was obtained in earlier work [17, 18] as given below,

$$\dot{P} = \frac{\gamma}{V}\left(\frac{P}{\rho}\dot{m} - P\dot{V}\right) \quad (3)$$

Active control strategies for a quarter car model of an active air suspension

The control strategy for this active air suspension is based on controlling the mass flow rate of air to the airspring. One major objective of a suspension is to reduce the transmissibility of suspension to zero. The active control system senses the response of the sprung mass to excitation from the ground and changes the mass flow rate of air to the air spring as a function of the velocity of sprung mass and/or unsprung mass. Earlier work [13] indicated that response to the excitation from the road in an active suspension with mass flow rate, as a function of velocity is lower than that of a suspension with mass flow rate as a function of the displacement of the sprung mass. Therefore, mass flow rate of air to the air spring is controlled by making it a function of velocity. The following three control strategies are evaluated

Strategy I - Mass flow rate is Function of relative velocity of sprung mass with respect to unsprung mass velocity

$$\dot{m} = \eta_1(\dot{x}_s - \dot{x}_u)$$

Strategy II - Mass flow rate is Function of velocity of unsprung mass $\dot{m} = \eta(\dot{x}_u)$

Strategy III -Mass flow rate is Function of velocity of sprung mass $\dot{m} = \eta_2(\dot{x}_s)$

The equation of motion and the mass flow relations are used to obtain the response for each of the proposed control strategies. It is assumed that the vehicle is under harmonic excitation from the road, which is defined as,

$$x_t = Xe^{i\omega t}$$

The response of the active suspension to harmonic excitation is obtained using the equations of motion and the mass flow conditions of air into the air spring. The response to harmonic excitation is assumed to harmonic. The response in bounce motion of sprung mass at and unsprung mass is given by x_s and $x_u$respectively. Where,

$$x_s = X_s e^{i\omega t} \text{ And } x_u = X_u e^{i\omega t}$$

7. Simulation of quarter car model of active suspension

The variation of transmissibility with excitation frequency is observed for all three active suspension control strategies mentioned earlier in this work.

The variation of amplitude ratio with respect to frequency is simulated. The air suspension parameters used in the simulation are as given in Table I. Equations of motion are used to obtain response equations. The response of passive system was simulated using parameters given in Table I. The two resonance frequencies of this system are 1.41Hz and 15.96 Hz.

Simulation is done for all active control using the response equations is given further in the paper and the most suitable strategy is proposed.

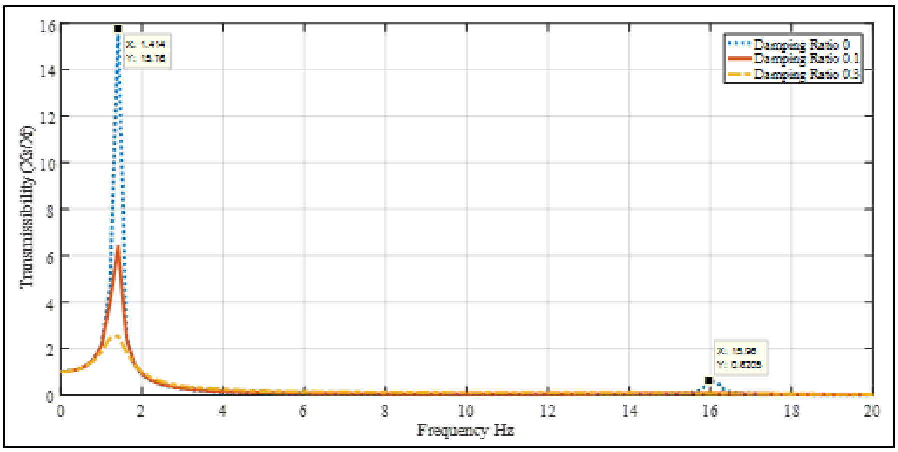

Fig 2: Response of passive suspension

Table 1: Suspension parameters used for simulation

Sprung mass	257.5 kg
Unsprung mass	20 kg
Tire Stiffness	181818.88 N/m
Excitation from ground	0.05 m
Specific gas constant (R)	287.05 J/kg-K
Temperature (T)	300°K

7.1 Strategy- I

Response equation when mass flow rate as function of difference between sprung mass velocity and unsprung mass velocity is given by

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{Bmatrix} X_s \\ X_u \end{Bmatrix} = \begin{bmatrix} 0 \\ k_t e^{i\omega t} X_r \end{bmatrix} \quad (4)$$

The coefficients a_{ij} etc. (where i, j vary from 1 to 2) are defined in Annexure.

The above model is a simulation for different values of gain and in the system. The quarter car model of active suspension is simulated using equation (4). The suspension parameter used are as given in Table 1. Amplitude ratio versus excitation frequency plot were obtained.

Mass flow rate as function of difference between sprung mass velocity and base velocity and negative values of gain

The effect of using the active control through the mass flow with the negative value of gain is to increase the stiffness of the air spring as well as the excitation to the air spring.

The excitation force acting on the mass and stiffness of the system both are doubled if the gain is -1. The effect of damping is to reduce the response of the system Fig. 3. Hence, we find that there is resonance in the system. The effect of damping is to decrease the amplitude ratio.

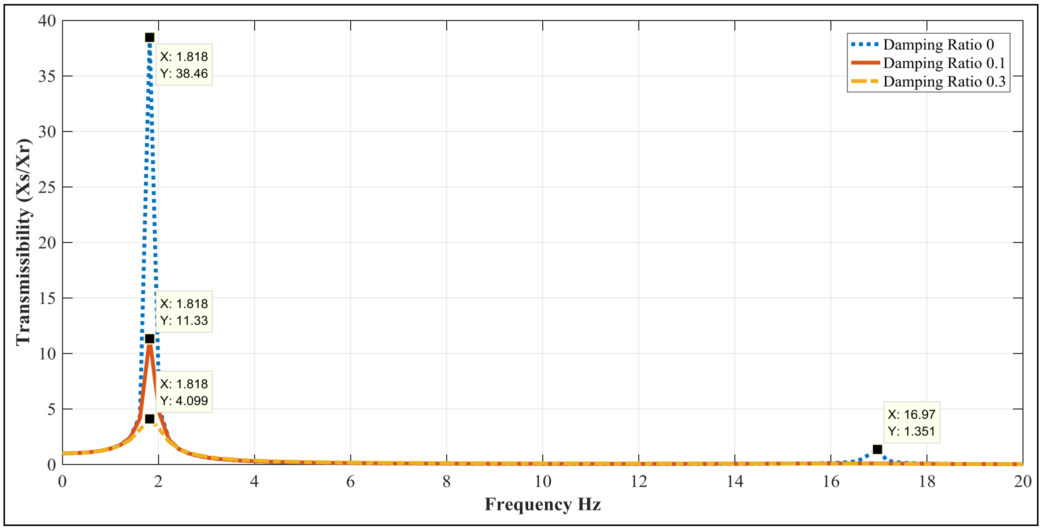

Fig 3: Response of an active mass flow rate as function of difference between sprung mass velocity and base velocity and gain equal to -1

Mass flow rate as function of difference between sprung mass velocity and base velocity and positive values of gain

The mass flow into air spring reduces the effective stiffness of the system as well as the total excitation on mass for positive values of gain. The air spring behaves as a negative spring for gain greater than 1. The excitation force due to mass flow into the air spring is opposite to excitation from the base passing through the ground, so the net excitation is reduced. The effective stiffness of the air spring is reduced to zero at gain equal to 1, Fig. 4. The system acts like a mass-damper system when the gain is equal to 1. Transmissibility is always less than 1 if the gain in the system is maintained at 1. Transmissibility is zero for an undamped system. It increases with increase in damping.

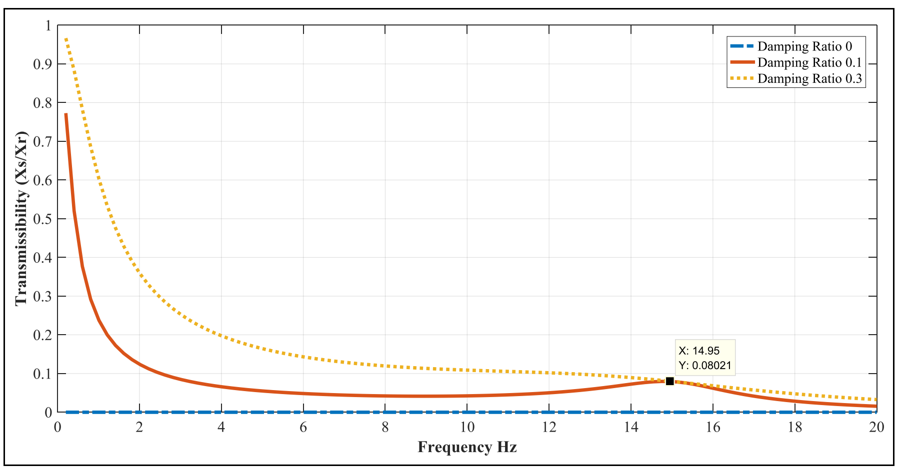

Fig 4: Response of an active mass flow rate as function of difference between sprung mass velocity and base velocity and gain equal to 1

7.2 Strategy- II

Response equation when mass flow rate as function of difference function of unsprung mass velocity is given by

$$\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{Bmatrix} X_s \\ X_u \end{Bmatrix} = \begin{bmatrix} 0 \\ k_t e^{i\omega t} X_r \end{bmatrix} \tag{5}$$

The coefficients b_{ij} (where i, j vary from 1 to 2) are defined in Annexure.

The above model is a simulation for different values of gain and in the system. The quarter car model of active suspension is simulated using equation (5). The suspension parameter used are as given in Table 1. Amplitude ratio versus excitation frequency plot were obtained.

Mass flow rate is function of unsprung mass velocity and positive values of gain

The gain in the active control system influences excitation force. But stiffness is not influenced by the gain in the control system. The transmissibility is higher than desired. The excitation due to the mass flow and excitation from the base acting through the spring act though the spring in the same direction. The transmissibility increases with increase in damping in the system. The transmissibility is doubled when gain is 1 in this system. It is seen in Fig 5 that the response of undamped system at low frequency is approaching infinity but gets reduced with increase in the damping in the system.

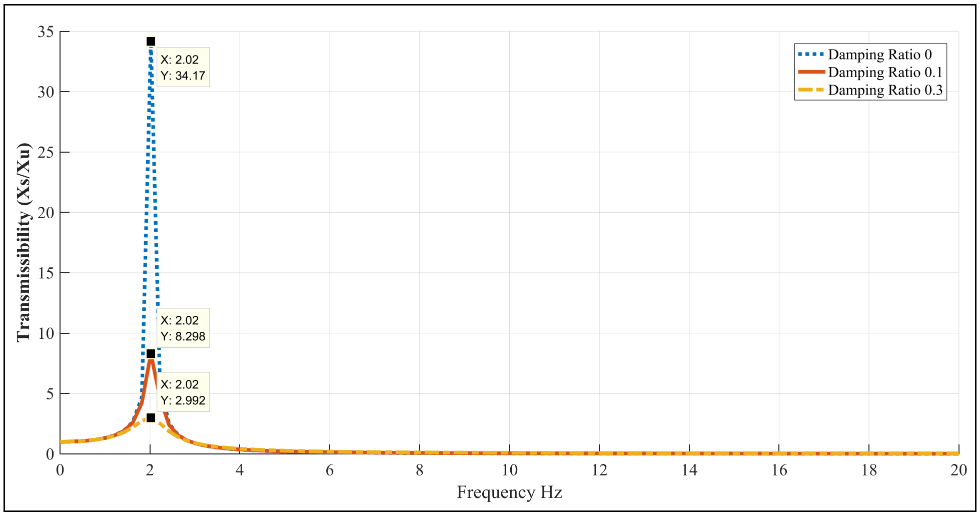

Fig 5: Response of a sprung mass, when active control is achieved by having mass flow rate is function of unsprung mass velocity and gain is 1.

Mass flow rate is function of unsprung mass velocity and negative values of gain

The excitation force due to mass flow rate acts in the opposite direction to the base excitation for negative values of gain. These results in the reduction in the total excitation passed to the sprung mass. Thus, the transmissibility of sprung mass too is lowered. The excitation due to mass flow cancels the excitation from the base passing through the spring if the gain is -1. It is seen in Fig. 6 that the response of an undamped system is zero. The transmissibility increases with increase in damping ratio.

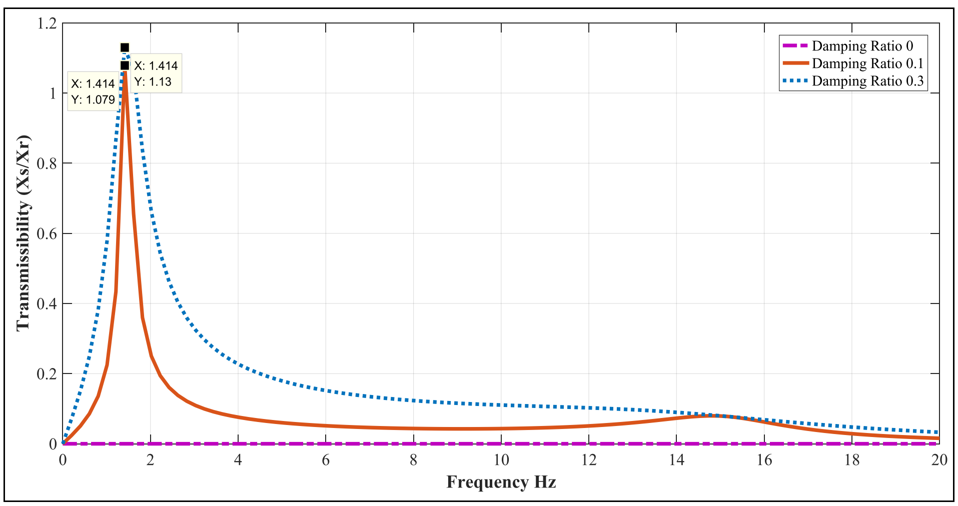

Fig 6: Response of a sprung mass, when active control is achieved by having mass flow rate is function of unsprung mass velocity and gain is -1

7.3 Strategy- III

Response equation when mass flow rate as function of difference function of sprung mass velocity is given by

$$\begin{bmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{bmatrix} \begin{Bmatrix} X_s \\ X_u \end{Bmatrix} = \begin{bmatrix} 0 \\ k_t e^{i\omega t} X_r \end{bmatrix} \qquad (6)$$

The coefficients q_{ij} (where i, j vary from 1 to 2) are defined in Annexure.

The above model is a simulation for different values of gain and in the system. The quarter car model of active suspension is simulated using equation (6). The suspension parameter used are as given in Table 1. Amplitude ratio versus excitation frequency plot were obtained.

Mass flow rate is a function of sprung mass velocity under positive values of gain

The total stiffness of the system is reduced for positive values of gain when Mass flow rate is a function of sprung mass velocity under positive values of gain. The system behaves as a negative spring for all positive values of gain.

The stiffness is zero for gain equal to 1 and the system behaves as a mass-damper system. High transmissibility at low frequency ratio but reduces rapidly to below 1. Fig. 7 indicates that the response of an undamped system is zero.

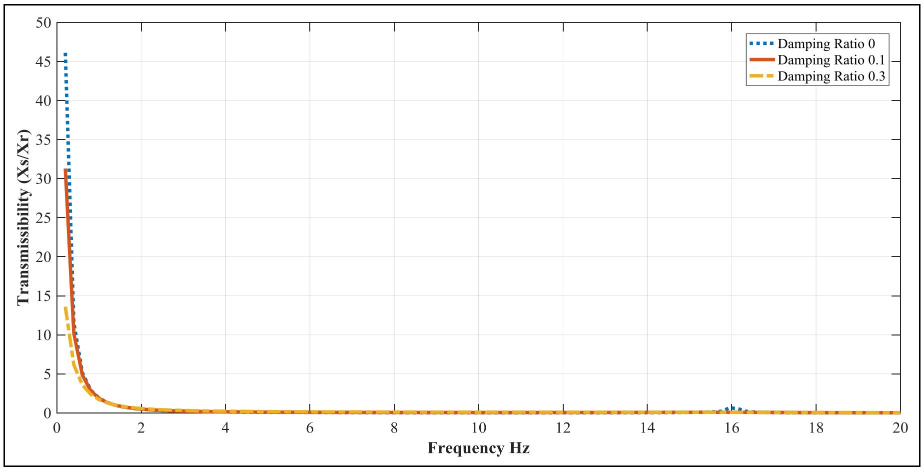

Fig 7: Response of a sprung mass when active control is achieved by having mass flow rate is function of sprung mass velocity and gain is 1

Mass flow rate is a function of sprung mass velocity under negative values of gain

In a system where the mass flow rate is a function of sprung mass velocity, the effect of negative values of gain is to increase the stiffness of the system. Hence, the transmissibility increases.

The resonance occurs at $\sqrt{2}\omega_n$ when gain is -1 Fig. 8. Transmissibility is influenced by damping. Transmissibility decreases with increase in damping in the system.

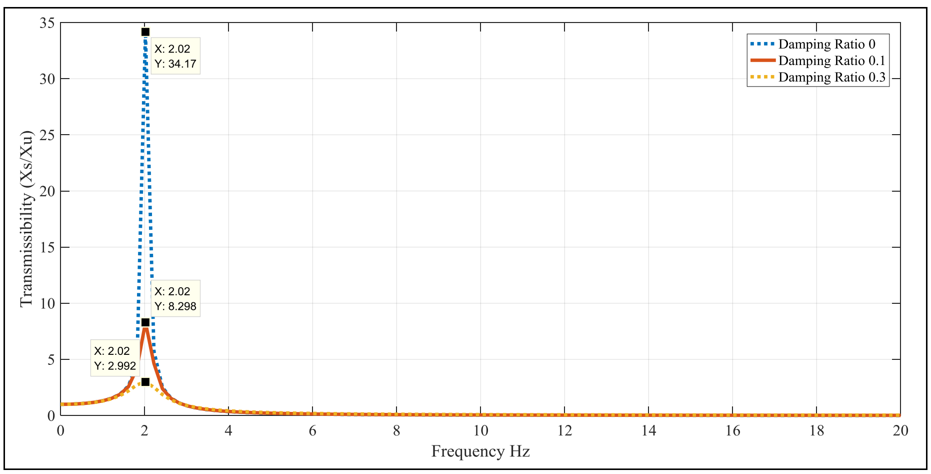

Fig 8: Response of a sprung mass when active control is achieved by having mass flow rate is function of sprung mass velocity and gain is 1

8. Discussions

The amplitude ratio is zero for an undamped pneumatic active suspension in which the active control achieved by having mass flow rate is a function of the difference between sprung mass velocity and base velocity for gain equal to 1. The undamped active pneumatic suspension system acts like a *mass-damper system* when the system has mass flow rate as a function of the difference between sprung mass velocity and base velocity for gain equal to 1 System using this active control strategy too, shows an increase in transmissibility as damping is increased. Transmissibility, in this case, does not exceed 1.

Amplitude ratio is also zero for an undamped pneumatic active suspension. Mass flow rate is a function of unsprung mass velocity. In this case, too the undamped active pneumatic suspension system acts like a *mass-damper system* when the system for gain equal to -1.

Therefore, the following two cases are recommended for use in active suspension:

- Controlling the mass flow rate of air to the airspring is recommended where the mass flow rate is a function of the relative velocity of the sprung mass and base and gain is 1.
- Controlling the mass flow rate of air is a function of the sprung mass velocity with gain is -1. The damping in the active control systems for both the control strategies suggested should be very low. This ensures that a separate damper isn’t required in the system.

9. Conclusions

Active control of air suspension by controlling mass flow rate makes it possible to dynamically change the characteristics of the air suspension corresponding to road input. It is concluded that the peak value of amplitude ratio is different for all the three strategies controlling the mass flow rate of air to the air suspension.

The peak value of transmissibility ratio for the bounce is zero for an undamped active pneumatic suspension using the either of the following two control strategies for specific values of gain.

- The Mass flow rate of air to airspring is function of the relative velocity of sprung mass to unsprung mass for gain 1.
- The Mass flow rate of air is a function of the sprung mass velocity with gain -1.
- For the aforementioned strategies, the stiffness becomes zero at specific values of gain, leading to system acting like a *mass damper system*. The peak value of transmissibility increases with increase in damping but does not exceed 1 for a small amount of damping in the system. Therefore, a separate damper is not needed in the system. The change in the stiffness of the suspension with mass flow into the airspring leads to changing the natural frequency of the system. Since, practically finding the relative velocity of sprung mass with respect to unsprung mass is feasible the active control of the pneumatic suspension by the making the mass flow of air to the airspring. The function of the relative velocity of sprung mass to unsprung mass is recommended. By choosing a suitable value of gain in the system, the characteristics of the air suspension get modified.

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