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# Analysis of queue with general vacations

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**Abstract**
Queueing systems in which the server works on primary and secondary customers have gained significance due to rapidly growing computer and communication networks. Here an M/M/2 queueing system with vacations having general distribution is considered where one of the two servers takes a vacation when there are no customers in the system. The equations determining the stationary distribution of the embedded Markov chain are provided. Iterative scheme for their solution is described. Vacation time distribution is considered as negative exponential and numerical results for probabilities and expected numbers of customers in the system are obtained. The behaviour of the results is in conformity with the results obtained by previous authors by a different technique. The advantage of the present scheme lies in its simplicity to apply on the model.

**Keywords:** M/M/2 Queueing system, vacations, Markov Chain

## Introduction

Queueing systems in which the server works on primary and secondary customers arise naturally as models of many computer, communication and production systems. As far as the primary customers are concerned, the server working on the secondary customers is equivalent to the server taking a vacation and not being available to the primary customers during this period. Thus, there is a natural interest in the study of queueing systems with server vacations. During the last thirty years, various researcher and practitioners have looked at models of this type, either to solve particular problems at hand or to develop understanding of the stochastic processes that arise from them. The techniques and results developed by these authors have been successfully used in a variety of applications. As discussed in the Chapters 5 and 6, there is an extensive literature on queues with vacations because of their wide applications in computer and communication systems. But most of the work is confined to single server. The reason is that the equations in case of more than one server become more intractable and closed form solutions become difficult.

Here an M/M/2 queueing system with vacations is considered where one of the two servers takes a vacation when there are no customers in the system. At the end of the vacation the server returns to the system. One server does not leave the system during other server's absence. The queue discipline is 'first come, first served'. The single vacation models for the M/G/1 queue were studied by Levy and Yechiali [1975] who observed the systems at epochs of service completion or vacation termination, and in order to distinguish between two instants considered two dimensional state spaces. The Markov Chains with transitions occurring at those instants are analysed. The N-policy vacation model for M/G/1 queue were analysed by Kella [1989]. In a single server models, only arrivals can occur during a server's vacation, while in multi-server models, service completions may also occur during a vacation period. M/G/1 models with various types of vacations are analysed by Fuhrman and Cooper [1985]. Zhang and Tian [2004] [8] have analysed a multi-server Markovian queueing model with vacations using matrix geometric method. Further Tian and Zhang [2006] have considered a multi-server Markovian queueing model with vacations and threshold policy.

## Mathematical Model

We consider a single vacation model. Here, when the system becomes empty one of the two servers takes exactly one vacation (even if no customers are in the system); on return from a vacation, the server stays and waits for customers to be served. The vacation times are independent and identically distributed non-negative random variables with a distribution function G(x) and a finite mean E(v). The model can be regarded as a combination of an M/M/2 queue and a M/M/1 queue. Therefore we construct the semi-regenerative process $\{X_t\}_{t\geq 0} = \{(\sigma_t, Q_t)\}_{t\geq 0}$ with the embedded Markov renewal process $\{X_n, T_n\}$, where $\sigma_t$ is defined in this model as

$$\sigma_t \equiv \begin{cases} 0, & if\ there\ are\ two\ servers\ in\ the\ system\ at\ time\,t, \\ 1, & if\ there\ is\ one\ server\ in\ the\ system\ at\ time\,t, \end{cases}$$

And $Q_t$ is the number of customers in the system just after time t. Let $T_n$ denotes epochs of arrival or service completion during a period when there are two servers in the system, and vacation termination, assuming $X_0 = (1,0)$. The state space of $\{X_n\}$ in this case is $S = \{(0,j) | j \geq 0\} \bigcup \{(1,0)\}$

The state (0, j) denotes that there are two servers and j customers in the system at an epoch of arrival or service completion. The state (1, 0) denotes that there are no customers and the vacation time starts.

The equations determining the stationary distribution $\{\pi_{i,j}\}$ of the embedded Markov chain $\{X_n\}$ are the following

$$\pi_{0,0} = a_0 \pi_{1,0}, \quad (7.1)$$

$$\pi_{0,1} = p\pi_{0,2} + a_1 \pi_{1,0}, \quad (7.2)$$

$$\pi_{0,2} = q_1 \pi_{0,1} + p\pi_{0,3} + a_2 \pi_{1,0}, \quad (7.3)$$

$$\pi_{0,j} = q\pi_{0,j-1} + p\pi_{0,j+1} + a_j \pi_{1,0}, \; j \geq 3, \quad (7.4)$$

$$\pi_{1,0} = p_1 \pi_{0,1}, \quad (7.5)$$

$$\sum_{j=0}^{\infty} \pi_{0,j} + \pi_{1,0} = 1, \quad \pi_{i,j} \geq 0, \; for\, all (i,j) \in S, \quad (7.6)$$

Where

$$p_1 \equiv \frac{\mu}{\lambda+\mu}, \; q_1 \equiv 1-p_1, \; p \equiv \frac{2\mu}{\lambda+2\mu}, \; q \equiv 1-p, \quad (7.7)$$

$$a_j \equiv \int_0^{\infty} b_{0,j}(t)\, dG(t), \quad (7.8)$$

$b_{0,j}(t)$ Being the probability that the number of customers present in the system at time *t* is *j* when no customers are in the system at time 0 in the ordinary M/M/1 queueing system with arrival rate λ and service rate μ.

**Method of solution**

The solution of the equations (7.1)-(7.6) can be obtained by applying the iterative scheme.

$$Let\, \pi_{0,1} = x \quad (7.9)$$

$$(7.5) \Rightarrow \pi_{1,0} = p_1 x \quad (7.10)$$

$$(7.1) \Rightarrow \pi_{0,0} = a_0 p_1 x \quad (7.11)$$

$$(7.3) \Rightarrow \pi_{0,2} = x\left[\frac{q_1}{p} + \frac{p_1}{p(1-\rho)}\left(-\rho + (1-a_0-a_1) + \rho a_1 + \rho a_0\right)\right]$$

$$(7.4) \Rightarrow \pi_{0,3} = x\left[\frac{q_1}{p}\rho + \frac{p_1}{p(1-\rho)}\left(-\rho^2 + (1-a_0-a_1-a_2) + \rho^2 a_1 + \rho a_2 + \rho^2 a_0\right)\right]$$

*Proceeding similarly and after simplification, we obtain*

$$\pi_{0,j} = x\left[\frac{q_1}{p}\rho^{j-2} + \frac{p_1}{p(1-\rho)}\begin{pmatrix} -\rho^{j-1} + (1-a_0-a_1-\ldots-a_{j-1}) \\ +\sum_{m=1}^{j-1} \rho^{j-m} a_m + \rho^{j-1} a_0 \end{pmatrix}\right] \quad (7.12)$$

$$for\; j \geq 2, \; where \;\; \rho = \frac{\lambda}{2\mu}$$

*Substituting* $\pi_{0,j}$ *in* (7.6),

$$x = \left[a_0 p_1 + \frac{q_1}{p(1-\rho)}\left(E(Y) - 1 + a_0\right) + p_1\right]^{-1} \quad (7.13)$$

Where Y denotes number of customers in the system at the end of vacation.

Obtaining $x$ by (7.13), values of probabilities may be obtained using (7.9)-(7.12). Once values of probabilities are known, expected length of the system is computed as

$$L_s = \sum_{j=0}^{\infty} j\pi_{0,j}$$

**Particular case**

We consider the vacation time distribution as negative exponential with parameter $\theta$.

For this distribution,

$$a_j = \frac{\theta \lambda^j}{(\theta+\lambda)^{j+1}}, \; j \geq 0 \quad (7.14)$$

$$E(Y) = \frac{1}{\theta} \quad (7.15)$$

This distribution was considered for the purpose of comparison of the results with Levy and Yechiali [1976] who applied decomposition technique for the same distribution.

**Numerical Results and Discussion**

To test the method of solution, extensive computations were carried out. First of all, vacation times distribution with parameter θ as 1 was considered. Values of $a_j$ given in (7.14) were substituted in equations (7.9)-(7.13). The equations were solved by the method given here, λ and μ were taken as 1 and 2 respectively so that ρ is 0.25. Then values of λ were changed to 2 and 3 respectively and entire procedure was repeated. Computational results for different probabilities and expected length of system for different values of ρ are reported in table 1. Values of the remaining probabilities are comparatively small and therefore not reported here. It is clear from the table that as values of ρ increase, average length of system $L_s$ also increases. The result is in conformity with observations of multi-server queueing model without vacations. For the purpose of sensitivity analysis, value of parameter θ was changed to 0.5 and 0.25 respectively corresponding to the same values of traffic intensity ρ. The results are reported in tables 2 and 7.3 respectively. The increase in values of $L_s$ with increase in ρ is evident from these tables also. Further more, as θ decreases i.e. mean of the vacation time distribution 1/θ increases, expected length of the system $L_s$ also increases. All these patterns are clearly visible from fig 1. This is in complete agreement with Igaki [1992] who has analysed exponential two server queue with single vacation where vacation time distribution is general. It is also observed from the tables that probabilities in all the cases increase upto a certain value and then starts decreasing. After certain number of customers, they become very small and it is due to this behaviour that computations of probabilities upto certain numbers of customers suffice to give fairly accurate values of performance measures.

**Conclusion**

Queueing systems in which the server works on primary and secondary (vacation) customers have gained importance due to their applications in computer and rapidly expanding communication systems. But most of the work is confined to single server queueing models with single or multiple vacations (Doshi [1986]). Multiserver queues with single or multiple vacations have received comparatively little attention. A two server queue with general vacation times distribution has been considered here. Simple iterative scheme is applied and probabilities are computed. With the purpose of numerical calculations, vacation times distribution is considered as negative exponential. Computational results are obtained for different values of traffic intensity ρ and parameter θ. The behaviour of the results is in complete agreement with the results of previous authors (Levy and Yechiali [1976], Igaki [1992]). The advantage of the scheme is its simplicity and ease with which it can be applied for the model.

**Table 1:** θ=1.0

| $\rho$ | $\pi_{0,0}$ | $\pi_{0,1}$ | $\pi_{0,2}$ | $\pi_{0,3}$ | $\pi_{0,4}$ | $\pi_{0,5}$ | $\pi_{1,0}$ | $L_s$ |
|---|---|---|---|---|---|---|---|---|
| 0.25 | 0.07 | 0.21 | 0.22 | 0.16 | 0.11 | 0.09 | 0.14 | 2.02 |
| 0.50 | 0.03 | 0.17 | 0.22 | 0.19 | 0.16 | 0.14 | 0.08 | 2.36 |
| 0.75 | 0.01 | 0.13 | 0.20 | 0.21 | 0.21 | 0.19 | 0.05 | 2.95 |

**Table 2:** θ=0.50

| $\rho$ | $\pi_{0,0}$ | $\pi_{0,1}$ | $\pi_{0,2}$ | $\pi_{0,3}$ | $\pi_{0,4}$ | $\pi_{0,5}$ | $\pi_{1,0}$ | $L_s$ |
|---|---|---|---|---|---|---|---|---|
| 0.25 | 0.05 | 0.21 | 0.23 | 0.17 | 0.14 | 0.06 | 0.14 | 2.04 |
| 0.50 | 0.01 | 0.14 | 0.22 | 0.20 | //0.18 | 0.16 | 0.07 | 2.70 |
| 0.75 | 0.01 | 0.12 | 0.20 | 0.21 | 0.21 | 0.20 | 0.05 | 2.99 |

**Table 3:** θ=0.25

| $\rho$ | $\pi_{0,0}$ | $\pi_{0,1}$ | $\pi_{0,2}$ | $\pi_{0,3}$ | $\pi_{0,4}$ | $\pi_{0,5}$ | $\pi_{1,0}$ | $L_s$ |
|---|---|---|---|---|---|---|---|---|
| 0.25 | 0.02 | 0.18 | 0.22 | 0.18 | 0.16 | 0.13 | 0.11 | 2.45 |
| 0.50 | 0.01 | 0.15 | 0.22 | 0.21 | 0.19 | 0.16 | 0.06 | 2.78 |
| 0.75 | 0.01 | 0.10 | 0.19 | 0.20 | 0.22 | 0.21 | 0.06 | 3.01 |

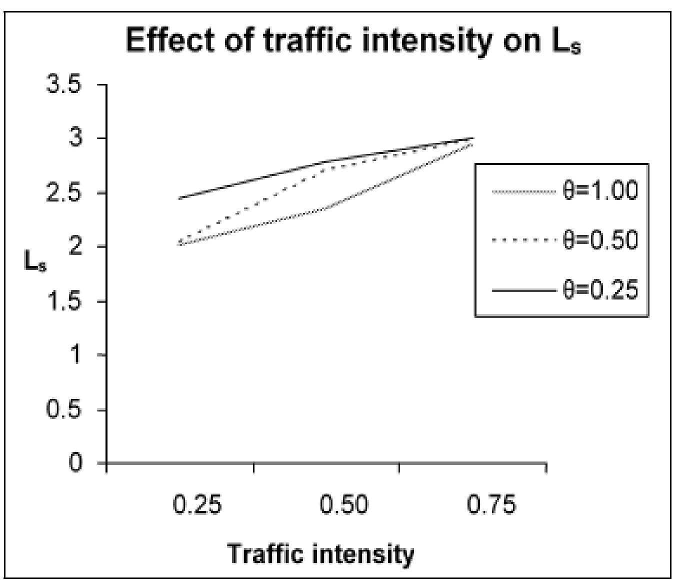


**Fig 1**

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