



## Labour policies in the DMP model: A theoretical analysis

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### Abstract

This paper discusses the results of a number of comparative statics exercises carried out in the context of the Diamond-Mortensen-Pissarides (DMP) matching model by distinguishing two alternative scenarios concerning the outflows from the labour market. The main results show that when the job-destruction rate is sufficiently influenced by the wage, labour market deregulation reduces wages but increases unemployment. This is counterintuitive to the standard result. In addition, endogenous redundancy exacerbates the reactivity of the labour market tightness indicator to productivity shocks.

**Keywords:** labour policies, job-destruction, labour market tightness, unemployment rate

### 1. Introduction

In the labour market workers search good jobs and firms also search good workers. The unemployed workers are absorbed in the vacant positions through matching with the firms. Production starts only when labour and firm are matched. Matching is not an instantaneous process, rather it is time-consuming and costly. Once there is a match in the labour market, both parties need not search for other and so search costs are saved and this gives birth to the surplus which is shared between the labour and the matched firm. The most commonly used surplus sharing rule is the Nash-bargaining rule. These features of the labour market have been conceptualized in the benchmark model of Diamond-Mortensen-Pissarides (called DMP hereafter).

The DMP model has been extended by introducing endogenous job-destruction, efficiency wage relation and business cycles. We find these extensions in the models of Albrecht *et al.* (1989) [2], Cole and Regerson (1989) [7], Mortensen and Pissarides (1994) [12], Abraham *et al.* (1995) [1], Zenou and Smith (1995) [20], Andolfatto (1996) [3], Shimer (2005) [17], Zenou (2005) [19], Arozamena and Ceneteno (2007) [5], Pissarides (2007) [15] etc.

Matching plays the central role in the DMP model. Matching is a function of unemployment rate and vacancy rate and is subject to CRS. The matching function has also been used by Hall (1979) [9], Pissarides (1979) [13], Diamond and Maskin (1979) [8], Bowden (1980) [6] etc. Mortensen (2011) [11] considers two types of matching function: linear and quadratic. Hosios (2003) [10] also shows that in efficiency worker's bargaining power is related to the elasticity of the matching function.

In this paper, we first describe the benchmark model of the DMP and then examine the consequences of different labour market policies on the Nash-wage rate, labour market tightness and on the unemployment rate in both the cases where job-destruction is exogenous as well as endogenous. Our comparative static results show that with exogenous redundancy, a policy of labour market deregulation is able to

reduce wages and unemployment at the same time. By contrast, when the job-destruction rate is sufficiently influenced by the wage, labour market deregulation reduces wages but increases unemployment. In addition, endogenous redundancy exacerbates the reactivity of the labour market tightness indicator to productivity shocks.

### 2. The Benchmark DMP Model

In the DMP model, job-matching between a job-seeker and a firm is expressed by the matching function:  $m = m(u, v)$ , where  $u$  is the unemployment rate and  $v$  is the vacancy rate in the labour market and  $m_1, m_2 > 0; m_{11}, m_{22} < 0$ . Total match flows is  $m = au$  and total job flows is  $m = vq$ . So, the job-

arrival rate is  $a = \frac{m}{u}$  and the job-offer rate is  $q = \frac{m}{v}$ .

Matching function is assumed to possess CRS property and

we can write  $a = \frac{m}{u} = \frac{m}{v} \cdot \frac{v}{u} = q(\theta) \cdot \theta$ , where  $\theta = \frac{v}{u}$  is the labour

market tightness and  $q'(\theta) < 0, \left| \frac{e_{q, \theta}}{q, \theta} \right| < 1$ .

The Bellman equations for unemployment ( $U$ ), employment ( $W$ ), vacancy ( $V$ ) and jobs filled in ( $J$ ) are

$$rU = b + \theta q(\theta)(W - U) \quad (1)$$

$$rW = w - \lambda(W - U) \quad (2)$$

$$rV = -C + q(\theta)(J - V) \quad (3)$$

$$rJ = y - w - rk - \lambda(J - V) \quad (4)$$

Where  $r$  is the discount rate,  $b$  is the unemployment benefit,  $C$  is the cost of maintaining vacancy,  $y$  is the constant match

productivity and  $q$  is the job offer rate,  $\lambda$  is the job-destruction rate,  $w$  is the wage rate and  $k$  is the capital hired per labour.

Using the zero-profit condition and using Equations (3), (4) we can write the steady state job creation condition as

$$y = w + rk + \frac{(r + \lambda)}{q(\theta)} C \tag{5}$$

The Nash-wage equation in the DMP model is

$$w = (1 - \beta)b + \beta(y + C\theta - rk) \tag{6}$$

Solving (5) and (6) one gets the equilibrium values of  $w, \theta$ .

The Beveridge curve is given by

$$u = \frac{\lambda}{\lambda + \theta q(\theta)} \tag{7}$$

Equilibrium  $u$  can be obtained from Equation (7) after determining equilibrium  $\theta$ .

### 3. Comparative Static Exercises

Taking total differentials of Equations (5), (6) and after simple manipulations one gets<sup>1</sup>

$$\left. \begin{aligned} \frac{\hat{W}}{\hat{y}} > 0, \frac{\hat{W}}{\hat{r}} < 0, \frac{\hat{W}}{\hat{\beta}} > 0, \frac{\hat{W}}{\hat{b}} > 0, \frac{\hat{W}}{\hat{\lambda}} < 0, \\ \frac{\hat{\theta}}{\hat{y}} > 0, \frac{\hat{\theta}}{\hat{r}} < 0, \frac{\hat{\theta}}{\hat{\beta}} < 0, \frac{\hat{\theta}}{\hat{b}} < 0, \frac{\hat{\theta}}{\hat{\lambda}} < 0 \end{aligned} \right\} \tag{8}$$

Again, taking total differentials of (7) and using (8) one gets

$$\frac{\hat{u}}{\hat{y}} < 0, \frac{\hat{u}}{\hat{r}} > 0, \frac{\hat{u}}{\hat{\beta}} > 0, \frac{\hat{u}}{\hat{b}} > 0, \frac{\hat{u}}{\hat{\lambda}} > 0 \tag{9}$$

The results obtained in (8) and (9) yield the following propositions:

**Proposition 1:** A positive productivity shock raises Nash-wage rate and labour market tightness but reduces unemployment rate.

**Proposition 2:** A high discount rate reduces wage rate and market tightness but raises unemployment rate.

**Proposition 3:** Labour market reforms (i.e. a lower  $\beta$ ) reduces wage rate and unemployment rate.

**Proposition 4:** A fall in unemployment benefit reduces wage rate and unemployment rate.

### 4. Endogenous Job-Destruction

Let us now assume that the job-destruction rate is endogenous and is inversely related to the wage rate. We may write

$$\lambda = \lambda(W), \lambda' < 0 \tag{10}$$

The well-settled literature on efficiency-wages would be able to provide a convincing rationale for the expression in (10) both on theoretical and empirical terms (e.g. Weiss 1991 and Ansar *et al.* 1997) [4].

Now taking total differentials of (5), (6), (7), (10) and after simplifications one gets

$$\left. \begin{aligned} \left(\frac{\hat{W}}{\hat{y}}\right)_{\lambda(W)} > 0, \left(\frac{\hat{W}}{\hat{r}}\right)_{\lambda(W)} < 0, \left(\frac{\hat{W}}{\hat{\beta}}\right)_{\lambda(W)} > 0, \left(\frac{\hat{W}}{\hat{b}}\right)_{\lambda(W)} > 0, \\ \left(\frac{\hat{\theta}}{\hat{y}}\right)_{\lambda(W)} > \left(\frac{\hat{\theta}}{\hat{y}}\right)_{\lambda(W)} > 0, \left(\frac{\hat{\theta}}{\hat{r}}\right)_{\lambda(W)} < \left(\frac{\hat{\theta}}{\hat{r}}\right)_{\lambda(W)} < 0, \left(\frac{\hat{\theta}}{\hat{\beta}}\right)_{\lambda(W)} < \frac{\hat{\theta}}{\hat{\beta}} < 0, \left(\frac{\hat{\theta}}{\hat{b}}\right)_{\lambda(W)} < \left(\frac{\hat{\theta}}{\hat{b}}\right)_{\lambda(W)} < 0 \end{aligned} \right\} \tag{11}$$

and

$$\left(\frac{\hat{u}}{\hat{y}}\right)_{\lambda(W)} < \left(\frac{\hat{u}}{\hat{y}}\right)_{\lambda(W)} < 0, \left(\frac{\hat{u}}{\hat{r}}\right)_{\lambda(W)} > \left(\frac{\hat{u}}{\hat{r}}\right)_{\lambda(W)} > 0,$$

and  $\left(\frac{\hat{u}}{\hat{\beta}}\right)_{\lambda(W)} < \dots > 0$  and  $\left(\frac{\hat{u}}{\hat{b}}\right)_{\lambda(W)} < \dots > 0$  according as  $e_{\lambda W} >, =, < \frac{W(1+e_{q,\theta})}{J(re_{q,\theta} - \lambda)}$  (12)

The above results lead to the following propositions:

**Proposition 6:** Market tightness and unemployment rate would be more volatile to changes in productivity and / discount rate if job-destruction is endogenous.

**Proposition 7:** Labour market deregulation under endogenous redundancy produces ambiguous effect on unemployment rate. However, if job-destruction rate is strongly associated with the wage rate then labour market deregulation increases unemployment rate. This is different from the case where job-destruction is exogenous.

The results presented in this paper can be easily tracked in a graphical manner. In the standard DMP model, the determination the equilibrium vacancy-unemployment pair, say  $(v, u)$ , occurs through the intersection of the Beveridge curve (BC) and the job-creation schedule (JC) (cf. Pissarides 2000) [14]. The former provides all the combinations vacancy-unemployment such that inflows in the labour market equal the corresponding outflows. The latter describes the non-predetermined decision of firms regarding vacancy opening for each level of unemployment. A possible equilibrium is illustrated in figure 1.

<sup>1</sup> See Appendix A.1.

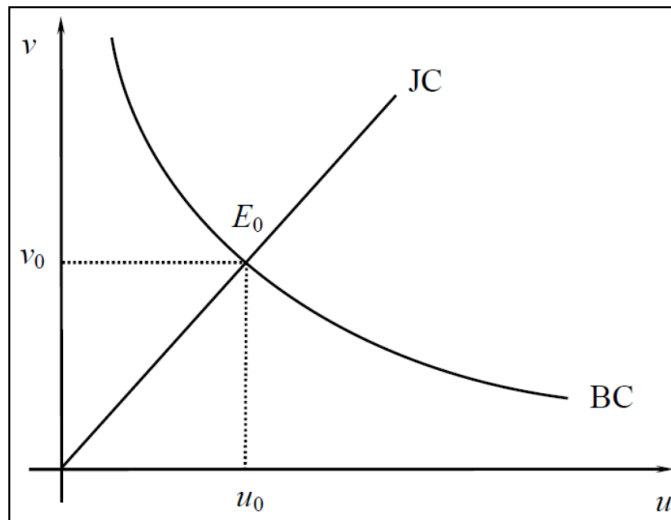


Fig 1: The equilibrium vacancy-unemployment pair

When the job-destruction rate, say  $\lambda$ , is exogenous, a policy of labour market deregulation, i.e., a reduction of the bargaining power of worker and/or a reduction of unemployment benefits, leads to a counter-clockwise rotation of the JC schedule. The intuition is that with lower wages, firms will find profitable to open more vacancies for each level of unemployment. Therefore, as it is illustrated in figure 2, the new equilibrium will be characterized by more vacancies and less unemployment fulfilling the objectives of policy makers.

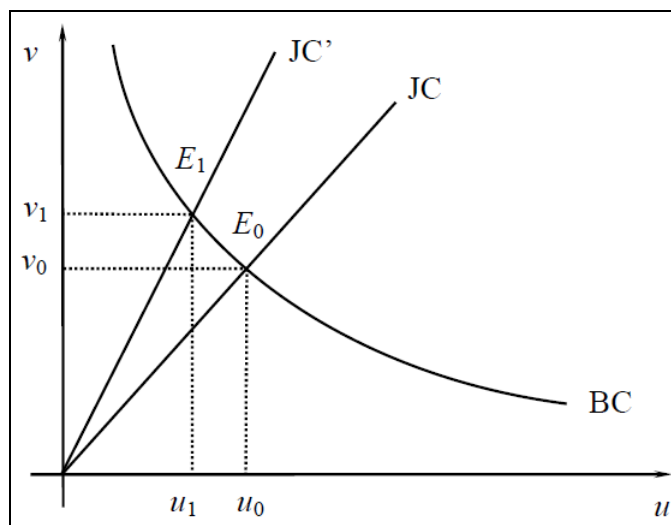


Fig 2: Labour market deregulation with exogenous redundancy

The effectiveness of labour market deregulation is less trivial when the job-destruction rate is endogenous. With regard to this, we consider the interesting case in which  $\lambda$  negatively depends on wages, say  $w$ .

The problematic effectiveness of labour market deregulation when  $\lambda$  is allowed to move in the opposite direction of  $w$  is due to the fact that when there is a reduction of the bargaining power of workers and/or a reduction of unemployment benefits, in addition to the counter-clockwise rotation of JC mentioned above, there is also an outward shift of the BC. As depicted in figure 3, we discuss the case in which this

additional movement of the BC hinders the increase in labour market tightness by leading to an increase in vacancies as well as in unemployment.

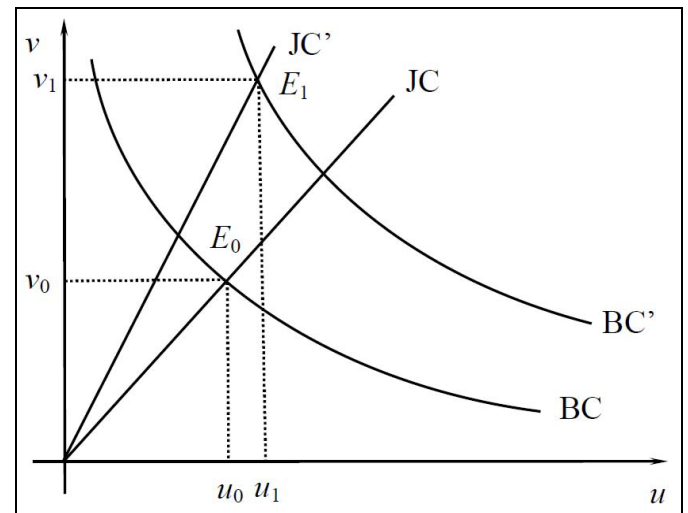


Fig 3: Labour market deregulation with endogenous redundancy

The situation illustrated in figure 3 is only one of the three possible outcomes;<sup>2</sup> indeed, it also can happen that labour market deregulation (i) increases vacancies by leaving unemployment unaltered or (ii) increases vacancies and reduce unemployment just as it happens in the standard case of figure 2.

### 5. Conclusion

Search and matching are the two basic features in the labour market. The path-breaking work in this line is the DMP model. This model analyses frictional unemployment in the matching framework in the labor market. In the benchmark DMP model, we find that if job-destruction is exogenous, a positive productivity shock raises Nash-wage rate and labour market tightness but lowers equilibrium rate of unemployment. A high discount rate reduces both the wage rate and the market tightness but raises unemployment rate. Both the Nash-wage rate and unemployment rate fall and market tightness rises if unemployment benefit is reduced. Further, a reduction in the worker's bargaining power lowers Nash-wage and unemployment rate but raises market tightness.

However, in reality it has been observed that job-destruction rate is more flexible than job-creation rate. Both empirical and theoretical works suggest negative association between job-destruction rate and wage rate. Our theoretical results establish that labour market tightness and equilibrium unemployment rate would be more volatile if job-destruction rate is inversely related to the wage rate. Further, labour market reforms may produce different effects on unemployment rate in the case of endogenous redundancy: (i) it may increase vacancies by leaving unemployment unaltered or (ii) increase vacancies and

<sup>2</sup> How strong have to be the link between  $\lambda$  and  $w$  in order to convey the ineffectiveness of labour market deregulation? Probably the JOLTS-based estimations presented by Shimer (2004, 2005) can provide some guidance to answer these important questions

reduce unemployment just as it happens in the case of exogenous redundancy and (iii) increase both vacancies and unemployment. The third case is possible only when the negative association between the job-destruction rate and the wage rate is very strong. These theoretical results highlight that the labour market deregulation with endogenous redundancy is not an unmixed blessing in the frictional labour market.

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