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A deterministic inventory model with demand & holding cost for weibull deteriorating items

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Abstract
This paper describes an EOQ inventory model when deterioration rate follows Weibull distributions. It is a two way parameter distribution of demand rate and other is holding cost. Which is assumed to be function of selling price and parabolic in terms of time respectively. In this EOQ model with shortage case is taken into consideration in detail. Whenever shortage allowed is completely backlogged. To achieve the result numerical examples for both cases shortage as well as without shortage are considered. The sensitive analysis for the inventory EOQ model has been performed to study the effect of changes the variation of parameters associated with the design model.

Keywords: deterministic inventory model, EOQ

1. Introduction

The inventory model of deteriorating items is interesting topic of current age of research. It is quite tough to discard the effect of deterioration, as it is very common feature of day to day life. In couple of decades many researchers have given theories which define deteriorating items. It explain about the items that become decayed, expired, invalid, damaged, devaluation and evaporative as time goes. Typically, it was considered that the items can preserve their characteristics while they kept stored in inventory. But it is not true for all cases. It is a big challenge in present scenario for decision makers to control and maintain the inventory of deteriorating items. The first economic order quantity (EOQ) inventory model was developed by Harris a century ago [1]. Later, it is generalized by Wilson to obtain formula for EOQ [2]. Within 1957 studied the deterioration of the fashion goods at the end of the prescribed shortage period. Couple of years later a model for an exponentially decaying inventory has been developed [3]. A model with Weibull deterioration rate without backordering is introduced in 1975. In ending of twentieth century lots of work has been done to design inventory model in this field like time varying demand, allowing of shortages and partial backlogging [4].

Recent trends in deteriorating item inventory modelling on the basis of demand variations and various other constraints like exponential declining demand and partial backlogging. A conscious efforts of Alamri & Balkhirevealed the effects of learning and forgetting on the optimal production lot size for deteriorating items with time-varying demand and deterioration rates [5]. The current age researchers find an optimized selling price (SP) and lot size with a broaden rate of deterioration and exponential partial backlogging [6]. They suppose that a fraction of consumers who backlog their orders increases exponentially as the waiting time for the next replenishment decreases. A deterministic inventory model has been introduced which describe that the deterioration rate is proportional to time [7]. Demand rate is also a time function of selling price (SP) & holding cost.

In the later section of this paper, EOQ model for deteriorating items has been explained, this model is based on some parameters: Weibull distribution, holding cost and demand rate. These parameters are depends on time [8]. Weibull distribution is expressed as parabolic functions of time, demand rate (DR) considered to be function of selling price (SP) and holding cost (HC) having similar functionality of weibull. In this paper an inventory model is developed for both cases shortage & without shortage.

2. EOQ Model

The elementary assumptions of this economic order quantity model are as follows:

1. The deterioration rate is proportional to time
2. The deterioration of units follows the two parameter Weibull distribution $\Psi(t) = \alpha.\beta.t^{\beta-1}$, where α is a shape parameter varying from 0 to 1.
3. Holding cost HC(t)per item per time- unit is time dependent and is assumed to be $HC(t) = HC + \delta.t^2$ where δ>0, HC>0
4. Shortage whenever allowed, are completely backlogged.
5. Demand rate is function of selling price.
6. Selling price (SP) follows an increasing trend, demand rate possess the negative derivative throughout its domain where demand rate is $f(s) = (a-s) > 0$
7. T is the time (length of cycle).
8. Replenishment is instantaneous and lead time is zero.
9. q is the order quantity in one cycle.

10. C is the order placing cost.
11. The selling price (SP) per unit item is denoted by s.
12. C1 is the unit cost of an item.
13. C2 is the shortage cost per unit per unit time.
14. The inventory holding cost (HC) per unit per unit time is HC (t).
15. Inventory is depleted due to deterioration as well as demand of an item. At time t1 the inventory becomes zero and shortage starts occurring.

Mathematical modelling concept of EOQ Model: The inventory level is denoted by Q (t) at time t ($0 \le t \le T$). The differential equations shown below is used to describe instantaneous state over the time interval of t.

$$\frac{dQ(t)}{dt} + \alpha\beta t^{\beta-1}Q(t) = -(a-s) \quad 0 \le t \le t_1 \tag{1}$$

$$\frac{dQ(t)}{dt} = -(a-s) \quad t_1 \le t \le T \tag{2}$$

With Q (t)=0 at $t=t_1$
By solving the above two equations and also neglecting higher powers of α, the desired equation is

$$Q(t) = (a-s)(1-\alpha t^{\beta})\{(t_1 - t) + \frac{\alpha}{\beta+1}(t_1^{\beta+1} - t^{\beta+1})\} \quad 0 \le t \le t_1 \tag{3}$$

and $$Q(t) = (a-s)(t_1 - t) \quad t_1 \le t \le T \tag{4}$$

Stock Loss: It can be occur due to deterioration& mathematically given by

$$SL = \int_0^t \alpha\beta t^{\beta-1}Q(t)dt = (a-s)\{\frac{\alpha t_1^{\beta+1}}{\beta+1}\} \tag{5}$$

Ordering Quantity: It is formulated by the following expression

$$q = SL + \int_0^T (a-s)dt \tag{6}$$

$$q = (a-s)T + (a-s)\frac{\alpha t^{\beta+1}}{\beta+1} \tag{7}$$

$$q = (a-s)[\frac{\alpha t^{\beta+1}}{\beta+1} + T] \tag{8}$$

Holding Cost: It is formulated by the following expression

$$HC = \int_0^t (HC + \delta t^2)Q(t)dt = \int_0^t (HC + \delta t^2)[(a-s)(1-\alpha t^{\beta})\{(t_1 - t) + \frac{\alpha}{\beta+1}(t_1^{\beta+1} - t^{\beta+1}\})]dt \tag{9}$$

Here, to simplify the above expression cubic and higher power of α has been neglected

$$HC = hc(a-s)[\frac{t_1^{\ 2}}{2} + \frac{\alpha\beta t_1^{\ \beta+2}}{(\beta+1)(\beta+2)} - \frac{\alpha^2 t_1^{\ 2\beta+2}}{2(\beta+1)^2}] + \delta(a-s)[\frac{t_1^{\ 4}}{12} + \frac{\alpha\beta t_1^{\ \beta+4}}{3(\beta+3)(\beta+4)} - \frac{\alpha^2 t_1^{\ 2\beta+4}}{(\beta+3)(2\beta+4)}] \tag{10}$$

Shortage Cost of Complete Cycle:

$$SC = \int_{t1}^{T} Q(t)dt \tag{11}$$

$$SC = (a-s)\frac{(t_1 - T)^2}{2} \tag{12}$$

Total profit per unit time is calculated using equation 5-12& described by

$$P(T, t_1, s) = s(a-s) - \frac{1}{T}(A + C_1 q + HC + C_2 SC) =$$

$$s(a-s) - \frac{1}{T}[A + C_1\{(a-s)\frac{\alpha t_1^{\ \beta+1}}{\beta+1} + (a-s)T\} + (a-s)\{h(\frac{t_1^{\ 2}}{2} + \frac{\alpha\beta . t_1^{\ \beta+2}}{(\beta+1)(\beta+2)}$$

$$- \frac{\alpha^2 \beta . t_1^{\ 2\beta+2}}{2(\beta+1)^2}) + \delta(\frac{t_1^{\ 2}}{12} + \frac{\alpha\beta . t_1^{\ \beta+4}}{3(\beta+3)(\beta+4)} - \frac{\alpha^2 . t_1^{\ 2\beta+4}}{(\beta+3)(2\beta+4)})\} + C_2\{\frac{1}{2}(a-s)(t_1 - T)^2\} \tag{13}$$

Here, Assumption has been taken: $t_1 = \gamma T$ where $0 < \gamma < 1$

$$P(T, s) = s(a-s) - \frac{1}{T}[A + C_1\{(a-s)\frac{\alpha(\gamma T)^{\beta+1}}{\beta+1}) + (a-s)T\} +$$

$$(a-s)\{h(\frac{(\gamma T)^2}{2} + \frac{\alpha\beta.(\gamma T)^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{\alpha^2\beta.(\gamma T)^{2\beta+2}}{2(\beta+1)^2}) +$$

$$\delta(\frac{(\gamma T)^2}{12} + \frac{\alpha\beta.(\gamma T)^{\beta+4}}{3(\beta+3)(\beta+4)} - \frac{\alpha^2.(\gamma T)^{2\beta+4}}{(\beta+3)(2\beta+4)})\} + C_2\{\frac{1}{2}(a-s)(\gamma-1)^2.T^2\} \tag{14}$$

Now profit can be calculated by using equation 14 which represents the profit function. In order to maximize the profit some necessary conditions will be applied on profit function:

$$\frac{\partial P(T,s)}{\partial T} = 0 \text{ and } \frac{\partial P(T,s)}{\partial s} = 0 \tag{15}$$

Sufficient condition to get maximum profit are

$$\frac{\partial^2 P(T,s)}{\partial T^2} < 0 \text{ and } \frac{\partial^2 P(T,s)}{\partial s^2} < 0 \tag{16}$$

After applying these boundary conditions we simplified the equation in the following manner

$$-\frac{A}{T^2}+C_1[\frac{(a-s)\alpha\gamma^{\beta+1}\beta T^{\beta-1}}{\beta+1}]+(a-s)[\frac{h\gamma^2}{2}+\frac{h\beta\gamma^{\beta+2}T^{\beta+1}}{\beta+2}-\frac{h\alpha^2(2\beta+1)\gamma^{2\beta+2}T^{2\beta}}{(\beta+1)^2}+..]$$
$$+C_2[\frac{1}{2}(a-s)(1-\gamma^2)=0 \quad (17)$$

and

$$(a-2s)-\frac{1}{T}[C_1\{-\frac{\alpha\gamma^{\beta+1}T^{\beta+1}}{\beta+1}-T]-h\{\frac{\gamma^2T^2}{2}+\frac{\alpha\beta\gamma^{\beta+2}T^{\beta+2}}{(\beta+1)(\beta+2)}-\frac{\alpha^2\beta\gamma^{2\beta+2}T^{2\beta+2}}{(\beta+1)^2}+..\}$$
$$-\delta\{\frac{\alpha(\beta+2)(T\gamma)^{\beta+4}}{(\beta+3)(\beta+4)}-C_2[T^2(1-\gamma^2)=0 \quad (18)$$

Using either MATLAB or Mathematica 5.1 tool eq. 17 & 18 has been solved. It conclude the optimized value of T* & S* respectively.

Problem (with shortage)

Assumption: A=200, a=100, C1=02, C2=1.2, h=0.4, α=0.1, β=0.3, γ=0.95, δ=0.1
In this problem input data is given to calculate profit by using eq.17 & 18:
P*(T,p)= 2174.12309, T* =2.99167, s* =60.364009, q=127.2031048, t1* =2.7698144

Problem (without shortage)

Assumption: A=200, a=100, C1=02, C2=1.2, h=0.4, α=0.1, β=0.3, γ=0.95, δ=0.1
P*(T,p)= 2174.08684, T* =3.000025, p* =60.36374, q=127.3504332, t1 * =2.77002375

3. Sensitive Analysis

All parameters which is studied in previous section playing important role in desired result. So in order to find that variations proper sensitive analysis is performed for this EOQ inventory model. 25% and 50% variation in parameters has been taken. For the purpose of EOQ sensitive analysis one parameter is changed and others remain the as usual. This concept of parameter variation shown in following table.

Table 1: represents the performance with shortage.

Parameters	Variation %	Profit (s)	Selling Price (S)	Time (T)	Ordering quantity (q)
A	-50	2232.855	61.1092	2.199	93.238
	-25	2195.1824	61.26213	2.739	116.378
	25	2167.0023	61.4407	3.252	138.249
	50	2141.5398	61.5196	3.551	150.987
a	-50	1374.3456	36.8075	4.513	66.025
	-25	1300.5042	51.4303	3.409	107.394
	25	3251.2493	71.2992	2.752	146.209
	50	5244.9656	86.0549	2.411	168.566
C1	-50	2249.6564	55.4629	3.082	149.836
	-25	2211.4013	59.3002	3.037	136.402
	25	2126.9308	63.0155	3.000	120.474
	50	2037.2088	66.2472	3.006	109.558
C2	-50	2173.1829	61.4027	3.018	128.224
	-25	2172.1962	61.3267	3.017	128.238
	25	2174.965	61.3564	3.016	128.174
	50	2174.435	61.4982	3.015	128.107
HC	-50	2171.8186	61.2515	3.231	137.869
	-25	2174.8426	61.3492	3.036	129.161
	25	2175.1561	61.4948	2.938	124.715
	50	2176.3326	61.4286	2.829	119.853
δ	-50	2167.6573	61.3202	3.277	139.567
	-25	2173.0943	61.3225	3.105	132.009
	25	2176.8294	61.3909	2.941	124.958
	50	2178.7823	61.3367	2.846	120.718
α	-50	2185.2713	60.9242	3.225	132.542
	-25	2179.1371	61.1922	3.098	129.931

	25	2171.0007	61.5486	2.937	126.434
	50	2164.9749	61.8853	2.823	123.799
β	-50	2169.6767	61.3197	3.260	138.148
	-25	2172.9341	61.3541	3.115	132.274
	25	2176.253	61.3811	2.917	124.155
	50	2179.1695	61.1857	2.771	117.145
γ	-50	2162.7708	60.9193	4.143	169.614
	-25	2166.9203	61.1751	3.621	151.164
	25	2175.8633	61.6658	2.509	107.701
	50	2164.8177	62.1308	1.961	85.358

Table 1 reveals some concluding remarks:

- If parameter A increase, it increases T*, s*, q* and decreases S*
- If parametera increase, it increases s*, q* and decreases T*, S*.
- If parameter C1increase, it increases s* and decreases T*, S*, q*.
- If parameter C2 increase, it increases s*,S* and decreases T*, q*.
- If parameter hincrease, it increases s*, S* and decreases T*, q*.
- If parameter δincrease, it increases s* and decreases T*, S* q*.
- If parameter αincrease, it increases s* and decreases T*, q*,S*.
- If parameter βincrease, it increases s* and decreases T*, q*,S*.
- If parameter γincrease, it increases s*, S* and decreases T*, q*.

4. Conclusion

In this paper we developed deterministic EOQ inventory model for deteriorating items with shortage case. The deterministic demand rate is a function of selling price. When shortage of items are considered, it is completely backlogged. Holding cost is also time dependent. Here, a good comparative analysis have been taken for parameters variations with shortage. In the numerical examples, it is concluded that the maximum average profit in without shortage case is more than that of the with shortage case. From the above EOQ inventory model one can calculate the optimum average profit margins for the shortage case and without shortage case for the deterministic inventory model with varying demand rate and holding cost subjected to the conditions.

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